Pf of Correctness

Let $\boxed{x_i \cdots x_j}$ be the max sum of $x_1 \cdots x_{n-1}$

$\boxed{x_k \cdots x_{n-1}}$ " " max suff sum "

Let $\boxed{x_a \cdots x_b}$ be max sum of $x_1 \cdots x_n$

Case 1, $b \leq n-1$. Then $x_a \cdots x_b$ is also OPT of $x_1 \cdots x_{n-1}$

So we must have $a = i$, $b = j$ ✓

Case 2: $b = n$. I claim $x_a \cdots x_{b-1}$ is max suffix sum of $x_1 \cdots x_{n-1}$

If not $\quad \underset{\substack{\| \\ x_{b-1}}}{x_a + \cdots x_{n-1}} < x_k + \cdots x_{n-1}$

$x_n + \cdots = x_b \quad\quad + x_n$

So this implies $x_a \cdots x_b$ is not OPT of $x_1 \cdots x_n$ Contradiction!

So this means $a = k$ and ALG correctly finds new OPT $\square$

We also need to prove new max suff sum.

So let $\boxed{x_a \cdots x_n}$ be new max suff sum of $x_1 \cdots x_n$

Case 1: OPT is empty. It must be that $x_k + \cdots x_n \leq 0$

So we correctly find OPT.

Case 2: OPT is non-empty We claim $x_a \cdots x_{n-1}$ is max suff subseq of $x_1 \cdots x_{n-1}$. If not then

$$\underbrace{x_n + x_a + \cdots x_{n-1}}_{\text{OPT = max suff}} < \underbrace{x_k + \cdots x_{n-1} + x_n}_{\text{another seq}}$$

Contradiction!

So we must have $k = a$, and we correctly update max suff seq $\square$
