Claim: Greedy gives a 2-approx for vertex cover.

Pf: Supp that greedy picks endpoints of edges $e_i \rightarrow e_k$.

So greedy chooses $2k$ vertexes.

(By definition it outputs a vertex cover).

Enough to show $\text{OPT} \leq k$.

Fact: $e_i \rightarrow e_k$ do not share endpoints.

B/C if $e_i$ and $e_j$ share endpoint (say $i < j$) then $e_j$ is already by $e_i$.

Fact: $\text{OPT} \leq k$. Every vertex in OPT can cover at most 1 of $e_i$-$e_k$. B/C they do not share endpoints, $\text{OPT} \leq k$.

\text{Lem:} Greedy gives $\log n$ approx for set cover.

\text{pf:} Assume $\text{OPT} = k$.

We want to show $\text{Greedy} \leq k \cdot \log n$.

Say $S_1$ is the first that we choose in Greedy.

I claim $|S_1| \geq \frac{n}{k}$. B/C OPT has $k$ sets.

So if a set in OPT covers $\frac{n}{k}$ elements (that is candidate for $S_1$.

So $S_1 \geq \frac{n}{k}$.

Rightarrow after first step # rm eleets $\leq n \left(1 - \frac{1}{k}\right)$.

I claim $S_2$ covers at least $\frac{n}{k}$ fraction of remaining eleets.

B/C OPT covers all remaining eleets by $k$ sets $\Rightarrow$ J set covering $\frac{n}{k}$ fraction.

Rightarrow after second step # rm eleets $\leq n \left(1 - \frac{1}{k}\right) \left(1 - \frac{1}{k}\right)$.

Repeat this.

After $t$ time # rm eleets $\leq n \left(1 - \frac{1}{k}\right)^t \leq n \cdot e^{-\frac{t}{k}}$. 

Let $t \approx \ln(n)$, $k = e^{-t} + e^{-\ln n} = \frac{1}{n}$.

So after $t$ iterations only 1 remains.

And we will be done by $k \cdot \ln n + 1$ sets.

So Greedy $\leq (c \cdot \ln n + 1 = O(\ln n))$ approx.