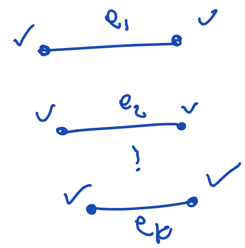
Claim. Greedy(2) gives a 2-approx for vertex cover.

__Pf__: Supp that greedy picks endpoints of edges $e_1 \dots e_k$.



So greedy chooses $2k$ vertexes.

(By definition it outputs a vertex cover).

Enough to show $OPT \geq k$.

__Fact__, $e_1 \dots e_k$ do not share endpoints.

B/C if e_i and e_j share endpoint (say $i<j$) then e_j is already by e_i

Fact $\Rightarrow OPT \geq k$. Every vertex in OPT can cover at most 1 of $e_1 - e_k$ B/C they do not share endpoints. So $OPT \geq k$

Lem: Greedy gives $\lg n$ approx for set cover.

__Pf__: Assume $OPT = k$

we want to show Greedy $\leq k \cdot \lg n$.

Say S_1 is the first that we choose in Greedy.

I claim $|S_1| \geq \frac{n}{k}$. B/C OPT has k sets. So $\exists$ a set in OPT covers n/k elmts (that is candidate for S_1.

So $S_1 \geq \frac{n}{k}$).

$\Rightarrow$ After first step # rem elmts $\leq n(1-\frac{1}{k})$.

I claim S_2 covers at least $\frac{1}{k}$ fract of remaining elmts

B/C OPT covers all remaining elmts by k sets $\Rightarrow \exists$ set covering $\geq \frac{1}{k}$ fraction.

$\Rightarrow$ after second step # rem elmts $\leq n(1-\frac{1}{k})(1-\frac{1}{k})$

; repeat this

after t time # rem elmts $< n(1-\frac{1}{k})^t \leq n \cdot e^{-\frac{1}{k} \cdot t}$

Let $t \geq (\ln n) \cdot k \Rightarrow e^{-\frac{1}{k} \cdot t} \leq e^{-\ln n} = \frac{1}{n}$

So after t iterations only 1 remains

and we will be done by $k \cdot \ln n + 1$ sets

So Greedy $\leq k \cdot \ln n + 1 \Rightarrow O(\lg n)$ approx.

$\boxed{1 - x \leq e^{-x}}$