$P(k) =$ " For any a, b s.t. $k\varepsilon \leq |a-b| \leq (k+1)\varepsilon$, and $\boxed{f(a) \cdot f(b) \leq 0}$ ALG finds an ε approx to a root of f ".

$P(1) = \varepsilon \leq a-b \leq 2\varepsilon$ Then there is a root output $\frac{(a+b)}{2}$.

IH: Assum $P(1)$ $P(2)$ -- $P(k-1)$ for some $k \geq 2$

equiv: for some $k \geq 2$, Assume $P(j)$ for all $1 \leq j \leq k-1$.

IS: Prove $P(k)$, Given arbitrary a, b s.t. $k\varepsilon \leq |a-b| \leq (k+1)\varepsilon$. Def: $m = \frac{a+b}{2}$.

Eval $f(m)$

- If $\boxed{f(a) \cdot f(m) \leq 0}$, then $|a-m| = |a - \frac{a+b}{2}| = |\frac{a}{2} - \frac{b}{2}| = \frac{1}{2}|a-b| \approx \frac{k}{2}\varepsilon$

So by $P(\frac{k}{2})$ on interval a, m. we find ε approx to a root.

- Otherwise $f(a) \cdot f(m) > 0$. Since $f(a) \cdot f(b) \leq 0$

$\Rightarrow \underbrace{f(a)^2}_{\geq 0} \underbrace{f(b) f(m)}_{\leq 0} \leq 0 \Rightarrow f(b) f(m) \leq 0$. by $P(k/2)$ on (m, b) we find ε approx to a root. □

IH ~~f(k)~~ Assum $P(k)$

IS: $P(2k)$

$\Rightarrow$ only prove $P(k)$ when $k = 2^{i}$ (only for pows of 2)

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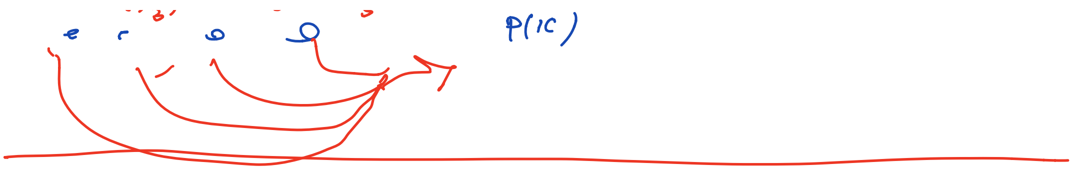
$P(1) \longrightarrow P(2) \longrightarrow P(3) \rightarrow$ - -

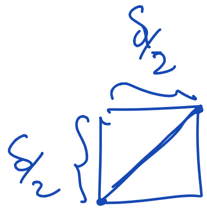
✓ ✓ ✓

$P(1)$ $P(2)$ $P(3)$ $P(4)$

✓

$P(k/2)$ $P(k/2)$ $P(\frac{2k}{3})$





$$\sqrt{\left(\sqrt{\delta/2}\right)^2 + \left(\sqrt{\delta/2}\right)^2} \approx 0.78.$$

Any pair of points in this square have distance $< \delta$