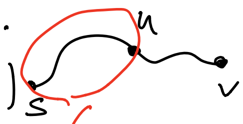
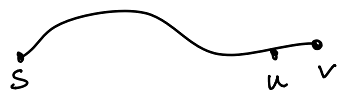
Correctness of Dijkstra.

Fact: If P_v is shortest path from s to v. u is on the path. Then the part from s to u (in P_v) is the shortest path from s to u.

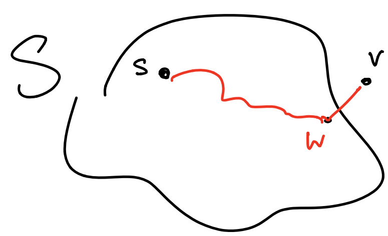




u is right before v.

If I know u, then first find P_u (shortest path to u) and then add (u,v).

In Dijkstra Assume we have found shortest p to S.

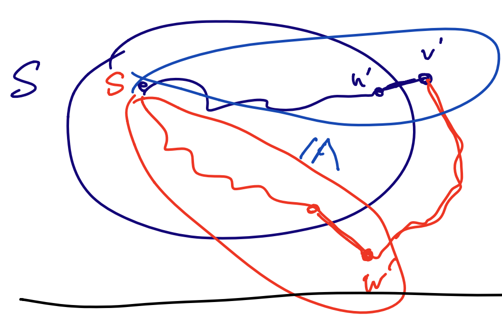


If I know the vertex before v is in S ⇒ ~~$P_v = P_u + (u,v)$~~

~~So just add v with this shortest path~~

$$P_v = \min_{w \in S} c(P_w) + c_{w,v}$$

Q: Find $v \notin S$, s.t. vertex before v in P_v (shortest path to v) is in S.



The candidate for v is the closest vertex to s.

Formal Pf. Induction. P(k) = Supp we have found shortest path to a S, where $|S| = k$.

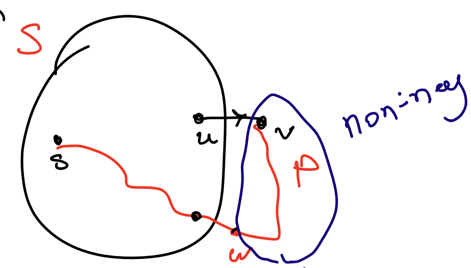
$P(1) = |S| = 1, \quad S = \{s\}$ ✓

IH: $P(k)$.

IS: Goal is to Prove $P(k+1)$. Assum that we have found shortest path to S s.t. $|S| = k$.

Assum v is the next vertex that we add, $d[v] = d[u] + c_{uv}$

Supp $c(P_v) < d[v]$ for contradiction



Let w be the first time shortest path P_v leaves S

$d[v] > c(P_v) \geq c(P_w) = d[w]$

cost of every edge ≥ 0

So $d[v] > d[w]$ contradiction for choosing v.

$$d[v] = \min_{u \in S} c(P_u) + c(u,v)$$
