Claim. Greedy is OPT for interval scheduling.

Pf:

Say Greedy chooses \( i_1, i_2 \ldots i_k \)

OPT = \( j_1, j_2 \ldots j_m \)

Since \( m \) is OPT we know \( m \geq k \).

So if \( |c| > m \Rightarrow m = k \downarrow \)

Lem: for all \( 1 \leq r \leq k \), \( f(i_r) \leq f(j_r) \)

- Greedy stays ahead method.

Pf of Lem: Use induction. \( P(r) = " f(i_r) \leq f(j_r)" \)

Base Case \( (r=1) \), first job \( i_1 \) has smallest finish time

So \( f(i_1) = \min f(\text{all jobs}) \leq f(j_1) \)

IH. Supp \( P(r) \)

IS. Prove \( P(r+1) \). We know \( f(i_r) \leq f(j_r) \) by IH.

Goal: \( f(i_{r+1}) \leq f(j_{r+1}) \)?

Job \( j_{r+1} \) is not yet chosen in Greedy.

Recall: \( i_{r+1} \) is the job with smallest finish time among all compatible with \( i_r \). \( i_r \Rightarrow j_{r+1} \) is an option

\[ \Rightarrow f(i_{r+1}) \leq f(j_{r+1}). \]

Lem: \( f(i_k) \leq f(j_k) \Rightarrow k \geq m \).

Suppose \( k < m \). Then OPT has a job \( j_{k+1} \)

But \( f(i_k) \leq f(j_k) \leq f(j_{k+1}) \).

So \( j_{k+1} \) starts after \( j_k \Rightarrow j_{k+1} \) is an option for greedy after \( i_k \). Contradiction. (Greedy must have added \( f(i_k), f(j_k), j_{k+1} \))
Claim: Greedy is OPT for Interval Partitioning.
Proof: Suppose Greedy allocates $d$-classrooms.
Goal: $d \leq \text{OPT}$. Recall $d \leq \text{OPT}$. 
Let $t$ be the time that classroom $d$ is allocated. 
All these jobs start before $t$. 
And, they end after $t$. 
Therefore at time $t+\epsilon$, $j$ is active, all of those $d-1$ jobs are active $\Rightarrow$ depth $\geq d$. 

\[ \text{Claim: Greedy is OPT for Interval Partitioning.} \]
\[ \text{Proof: Suppose Greedy allocates } d \text{-classrooms.} \]
\[ \text{Goal: } d \leq \text{OPT}. \text{ Recall } d \leq \text{OPT}. \]
\[ \text{Let } t \text{ be the time that classroom } d \text{ is allocated.} \]
\[ \text{At that time all other } d-1 \text{ classrooms are allocated to other jobs.} \]
\[ \text{All these jobs start before } t. \]
\[ \text{B/C Greedy starts by selecting these jobs.} \]
\[ \text{And, they end after } t. \]
\[ \text{Therefore at time } t+\epsilon, j \text{ is active, all of those } d-1 \text{ jobs are active } \Rightarrow \text{ depth } \geq d. \]