Claim. Greedy is OPT for interval scheduling.

Pf:

Say Greedy chooses $i_1 \; i_2 \; \cdots \; i_k$

OPT " $j_1 \; j_2 \; \; j_m$

Goal: $k \geq m$ $\Leftrightarrow$ $k = m$ later

BC m is OPT we know $m \geq k$.

So if $\boxed{k \geq m} \Rightarrow m = k$ ✓

Lem: for all $1 \leq r \leq k$, $f(i_r) \leq f(j_r)$

– Greedy stays ahead method.

Pf of Lem: Use induction. $P(r) =$ " $f(i_r) \leq f(j_r)$ ".

Base Case $(r=1)$. first job i_1 has smallest finish time

So $f(i_1) = \min f(\text{all jobs}) \leq f(j_1)$

IH. Supp $P(r)$

IS. Prove $P(r+1)$. We know $f(i_r) \leq \underbrace{f(j_r)}_{f(j_r) \leq f(j_{r+1})}$ by IH.

Goal: $f(i_{r+1}) \leq f(j_{r+1})$?

Job j_{r+1} is not yet chosen in Greedy.

Recall: i_{r+1} is the job with smallest finish time among all compatible with $\{i_1 \cdots i_r\} \Rightarrow j_{r+1}$ is an option

$\Rightarrow f(i_{r+1}) \leq f(j_{r+1})$.

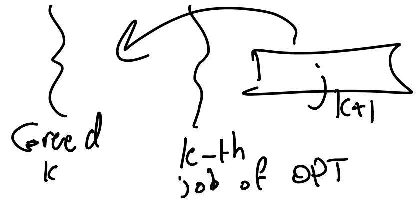
Lem: $f(i_k) \leq f(j_k) \overset{?}{\Rightarrow} k \geq m$.

Suppose $k < m$. Then OPT has a job j_{k+1}

But $f(i_k) \leq f(j_k) \leq f(j_{k+1})$.

So j_{k+1} starts after $j_k \Rightarrow j_{k+1}$ is an option for greedy after i_k contradiction (Greedy must have added j_{k+1})

$f(i_k)$ $f(j_k)$



Claim: Greedy is OPT for Interval Partitioning.

Pf: Suppose Greedy allocates d-classrooms.

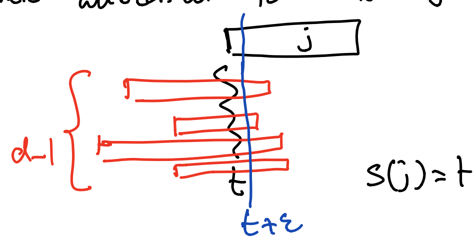
Goal: $d \leq OPT$. Recall depth $\leq OPT$.

$d \leq$ depth $\Rightarrow$ $d \leq OPT$.

Let t be the time that classroom d is allocated.

At that time all other $d-1$ classroms were allocated to other jobs

All $\underbrace{\text{these jobs start before } t}_{\text{B/C Greedy sorts by starting time}}$



And, they end after t.

Therefore at time $t+\varepsilon$, j is active, all of those $d-1$ jobs are active $\Rightarrow$ depth $\geq d$.