For Induction.
\( P(n) \): Set \( \{1, \ldots, n\} \) has exactly \( 2^n \) subsets.

**Goal:** prove \( P(n) \) for all \( n \geq 1 \).

**Base Case:** \( P(1) \). Set \( \{1\} \) two subsets \( \emptyset, \{1\} \)

\( \text{IH:} \ P(n-1) \) holds.

\( \text{IS:} \ \text{Goal: prove } P(n) \).

\( \text{God:} \ \{1, \ldots, n\} \) has \( 2^n \) subsets.

By \( \text{IH} \), \( \{1, \ldots, n-1\} \) has \( 2^{n-1} \) many subsets.

Every subset \( S \) of \( \{1, \ldots, n\} \)

I can construct two subsets of \( \{1, \ldots, n\} \). \( S, S \cup \{n\} \).

\( \checkmark \) Observe that this way I construct every subset of \( \{1, \ldots, n\} \).

\( \rightarrow \) By \( \text{BC} \) any subset \( T \subseteq \{1, \ldots, n\} \), \( T = \{n\} \subseteq \{1, \ldots, n\} \)

\( \checkmark \) I construct every subset at most once.

\( \{1, \ldots, n\} \) has twice many subsets, i.e. \( 2 \cdot 2^{n-1} = 2^n \) \( \square \)

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**Claim:** You can change any amount of postage \( \geq 12 \) using 4-5 cents stamps.

\( P(n) \): \( n \) can be changed using 4-5 cents stamps.

**Base Case:** \( P(12), P(13), P(4) \) \( P(15) \)

\( 4+4+4 \quad 5+4+4 \quad 5+5+4 \quad 5+6+4 \)

\( \text{IH:} \ P(k) \) holds for all \( 1 \leq k \leq n-1 \), and \( n \geq 16 \)

\( \text{IS:} \ P(n) \).

pay a 4-cents stamp. we use \( \text{IH} \) to pay for \( P(n-4) \).