For Induction.

$P(n)$ = Set $\{1, -, n\}$ has exactly 2^n subsets.

Goal: prove $P(n)$ for all $n \geq 1$.

Base Case: $P(1)$. Set $\{1\}$ two subsets $\emptyset, \{1\}$

IH: $p(n-1)$ holds.

IS. Goal: prove $p(n)$.

Goal: $\{1, -, n\}$ has 2^n subsets.

By IH. $\{1, -, n-1\}$ has 2^{n-1} many subsets.

Evry subset S of $\{1, -, n-1\}$

I can construct two subsets of $\{1, -, n\}$. S, $S \cup \{n\}$.

✓ Observe that this way I construct evry subset of $\{1, -, n\}$.

↳ BC evry subset $T \subseteq \{1, -, n\}$, $T - \{n\} \subseteq \{1, -, n-1\}$

✓ I construct evry subset at most once.

$\{1, -, n\}$ has twice many subsets. i.e. $2 \cdot 2^{n-1} = 2^n$ ☐

Claim: You can chang any amount of postag ≥ 12 using 4-5 cents stamps.

$P(n)$: n can be changed using 4-5 cents stamps.

Base Case: $P(12)$, $P(13)$ $P(14)$ $P(15)$

$P(12) \to 4+4+4$, $P(13) \to 5+4+4$, $P(14) \to 5+5+4$, $P(15) \to 5+5+5$

IH: $P(k)$ holds for all $12 \leq k \leq n-1$, and $n \geq 16$

IS: $P(n)$.

Pay a 4-cents stamp. we use IH to pay for $p(n-4)$.