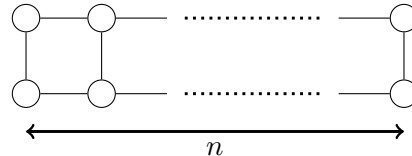


Homework 6

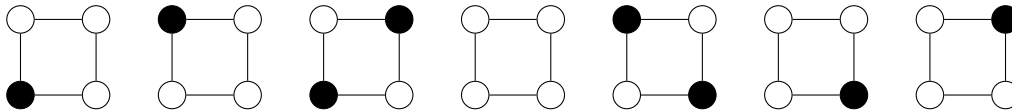
Shayan Oveis Gharan

Due: May 21, 2020 at 23:59 PM

- P1) Given the following graph with $2n$ vertices. Design a dynamic program that runs in polynomial time and outputs the number of independent sets of this graph. For example, if $n = 2$ you



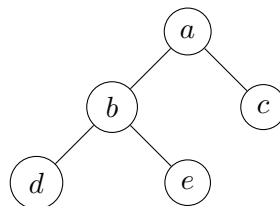
should output 7. See below for all these 7x independent sets when $n = 2$,



- P2) You have just learned wall climbing. The wall that you are practicing on is $n + 1$ squares tall and $k + 1$ squares wide. The bottom left square is at $(0, 0)$ and the top right is at (k, n) . There are two ladders on this wall one is from $(z_1, 0)$ all the way to (z_1, n) and the other from $(z_2, 0)$ all the way to (z_2, n) . You can assume $0 \leq z_1 < z_2 \leq k$.

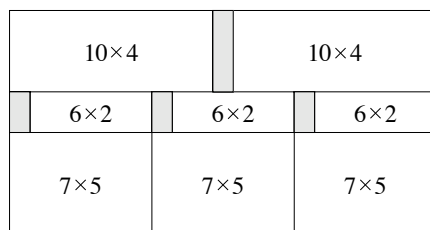
Your friend has hanged two coins in every row of this wall; say the coins at row i are located at $(x_i, i), (y_i, i)$ where $0 \leq x_i < y_i \leq k$ for all i . You start at $(0, 0)$ and you want to collect all coins as fast as possible. In each step, you either go left, go right, or if you are on the ladder you can go up, but if you go up you cannot come back down; each of these moves take 1 time-step. Design an algorithm that runs in time polynomial in n, k and outputs the smallest number of time-steps you need to collect all coins.

- P3) You are given a tree T where every node i has weight $w_i \geq 0$. Design a polynomial time algorithm to find the weight of the smallest vertex cover in T . For example, suppose in the following picture $w_a = 3, w_b = 1, w_c = 4, w_d = 3, w_e = 6$. Then, the minimum cost vertex cover has nodes a, b with weight $3 + 1 = 4$.



- P4) Suppose we have received a large rectangular marble slab of size $W \times H$ and we want to cut the slab to obtain rectangular marble plates of sizes $W_1 \times H_1, W_2 \times H_2, \dots, W_n \times H_n$. Any

piece of marble (the slab or the plates cut from it) can be cut either horizontally or vertically into two rectangular plates with integral widths and heights, cutting completely through that piece. This is the only way to cut pieces and pieces cannot be joined together.



Since the marble has a pattern on it, the plates cannot be rotated: if a plate of size $A \times B$ is cut, then it cannot be used as a plate of size $B \times A$ unless $A = B$. We can make zero or more plates of each desired size. A marble plate is wasted if it is not of any of the desired sizes after all cuts are completed. The question is how to cut the initial slab so that as little of it as possible will be wasted.

As an example, assume that original slab is 21×11 and the desired plate sizes are 10×4 , 6×2 , 7×5 , and 15×10 . The minimum possible area wasted is 10, and the figure (below) shows one sequence of cuts with total waste area of size 10. Design an algorithm that runs in time polynomial in n, W, H that outputs the minimum possible waste area.

P5) **Extra Credit:** Given a sequence of positive numbers x_1, \dots, x_n and an integer k , design a polynomial time algorithm that outputs

$$\sum_{S \in \binom{[n]}{k}} \prod_{i \in S} x_i,$$

where the sum is over all subsets of size k .