Please see https://courses.cs.washington.edu/courses/cse421/18wi/grading.html for general guidelines about Homework problems.

Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers.

P1) (20 points) Let $G$ be a tree. Use induction to prove that we can color vertices of $G$ with 2 colors such that the endpoints of every edge have opposite colors. For example, in the following picture we give a coloring of the given tree with black and white.

P2) (20 points) Given a connected undirected graph $G = (V, E)$, design an $O(m+n)$-time algorithm to find a vertex in $G$ whose removal does not disconnect $G$. Note that as a consequence this algorithm shows that every connected graph contains such a vertex.

P3) (25 points) Given a connected graph $G = (V, E)$ with $n$ vertices and $m$ edges.

a) (10 points) Design an $O(m+n)$ time algorithm that outputs the vertices of a cycle of $G$. If $G$ has no cycles, your algorithm should output “no cycle”. For example, given the following graph your algorithm may output 1, 2, 4, 3, 1.

b) (10 points) We say a tree $T$ is a spanning tree of $G$ if $T$ has $n-1$ edges and edges of $T$ are a subset of $E$. Suppose that $m \geq n$. Prove that $G$ has at least 3 spanning trees. For example, the following are 3 spanning trees of the above graph (note that the above graph has 7 spanning trees):

c) Given a connected graph $G = (V, E)$ with $m \geq n$, design an $O(m+n)$ time algorithm that outputs three spanning trees of $G$. For example, in the above examples, your algorithm may output as follows (note that the format of the output is not important):

- $(1, 2), (2, 3), (3, 4)$
- $(1, 2), (2, 3), (2, 4)$
- $(1, 2), (1, 3), (3, 4)$
P4) (20 points) Given a graph $G = (V, E)$ with $n$ vertices such that the degree of every vertex of $G$ is at most $k$. Furthermore, assume that in every connected component of $G$ there is a vertex of degree smaller than $k$. Show that we can color vertices of $G$ with $k$ colors such that the endpoints of every edge of $G$ have distinct colors. See the following graph for an example (here $k = 3$)

P5) **Extra Credit:** Prove that we can color the edges of every graph $G$ with two colors (red and blue) such that, for every vertex $v$, the number of red edges touching $v$ and the number of blue edges touch $v$ differ by at most 2.