


CSE 421 Algorithms

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Lecture 24

Network Flow Applications

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Today's topics

- Image Segmentation
- Strip Mining
- Reading: 7.5, 7.6, 7.10-7.12

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Minimum Cut Applications

- Image Segmentation
- Open Pit Mining / Task Selection Problem
- Reduction to Min Cut problem

S, T is a cut if S, T is a partition of the vertices with s in S and t in T

The capacity of an S, T cut is the sum of the capacities of all edges going from S to T

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Separate Lion from Savana

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Simplified version from Wednesday

Image analysis

- a_i: penalty of assigning pixel i to the foreground
- b_i: penalty of assigning pixel i to the background
- p_{ij}: penalty for assigning i to the foreground, j to the background or vice versa
- A: foreground, B: background
- $Q(A,B) = \Sigma_{\{i \text{ in } A\}} a_i + \Sigma_{\{j \text{ in } B\}} b_j + \Sigma_{\{(i,j) \text{ in } E,\ i \text{ in } A,\ j \text{ in } B\}} p_{ij}$
- Minimize Q(A,B)
- Assume foreground/background penalties are either infinite or zero
  - So they just require some pixels to be foreground/background

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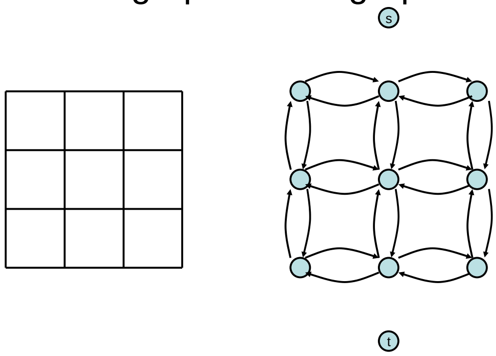
Pixel graph to flow graph



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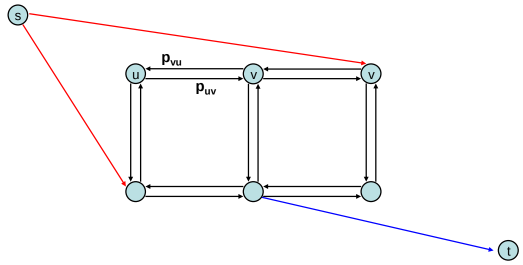
Mincut Construction



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Open Pit Mining

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Application of Min-cut

- Open Pit Mining Problem
- Task Selection Problem
- Reduction to Min Cut problem

S, T is a cut if S, T is a partition of the vertices with
s in S and t in T
The capacity of an S, T cut is the sum of the capacities of
all edges going from S to T

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Open Pit Mining

- Each unit of earth has a profit (possibly negative)
- Getting to the ore below the surface requires removing the dirt above
- Test drilling gives reasonable estimates of costs
- Plan an optimal mining operation

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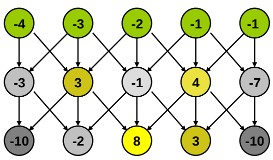
Mine Graph



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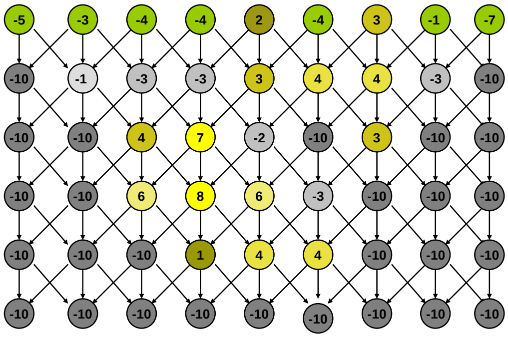
Determine an optimal mine



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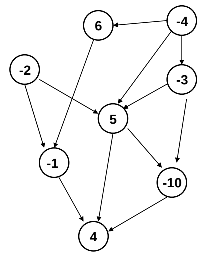
Generalization

- Precedence graph G=(V,E)
- Each v in V has a profit p(v)
- A set F is *feasible* if when w in F, and (v,w) in E, then v in F.
- Find a feasible set to maximize the profit



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Min cut algorithm for profit maximization

- Construct a flow graph where the minimum cut identifies a feasible set that maximizes profit

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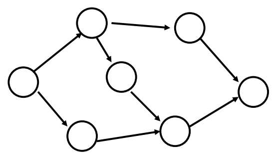
Precedence graph construction

- Precedence graph G=(V,E)
- Each edge in E has infinite capacity
- Add vertices s, t
- Each vertex in V is attached to s and t with finite capacity edges



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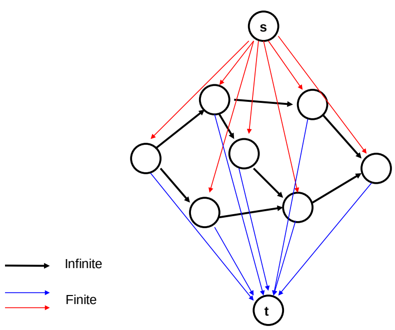
Find a finite value cut with at least two vertices on each side of the cut



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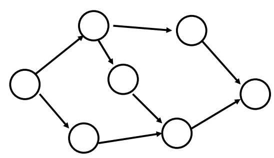
The sink side of a finite cut is a feasible set

- No edges permitted from S to T
- If a vertex is in T, all of its ancestors are in T



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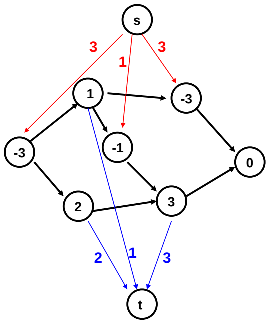
Setting the costs

- If p(v) > 0,
  - cap(v,t) = p(v)
  - cap(s,v) = 0
- If p(v) < 0
  - cap(s,v) = -p(v)
  - cap(v,t) = 0
- If p(v) = 0
  - cap(s,v) = 0
  - cap(v,t) = 0



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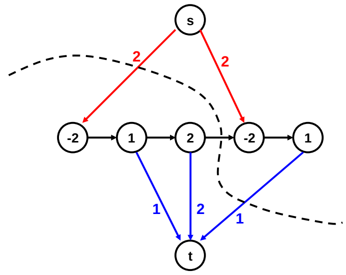
Minimum cut gives optimal solution Why?



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Computing the Profit

- Cost(W) = $\Sigma_{\{w \text{ in } W;\ p(w) < 0\}}$-p(w)
- Benefit(W) = $\Sigma_{\{w \text{ in } W;\ p(w) > 0\}}$ p(w)
- Profit(W) = Benefit(W) – Cost(W)

- Maximum cost and benefit
  - C = Cost(V)
  - B = Benefit(V)

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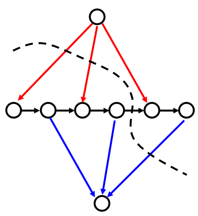
Express Cap(S,T) in terms of B, C, Cost(T), Benefit(T), and Profit(T)



Cap(S,T) = Cost(T) + Ben(S) = Cost(T) + Ben(S) + Ben(T) – Ben(T)
= B + Cost(T) – Ben(T) = B – Profit(T)

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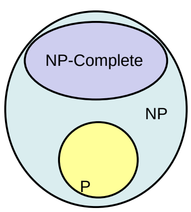
The Theory of NP-Completeness



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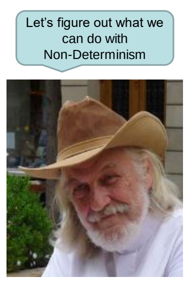
Jack Edmonds



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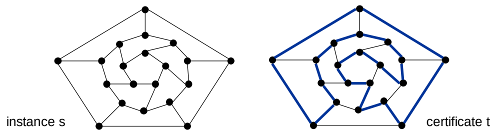
Non-deterministic polynomial time

- Problems where "yes" instances have polynomial time checkable certificates



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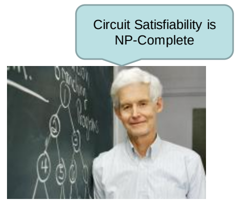
Steve Cook



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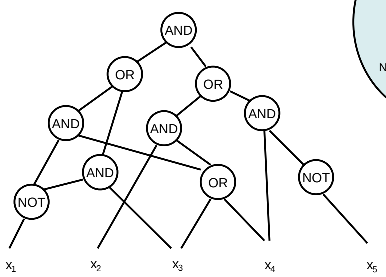
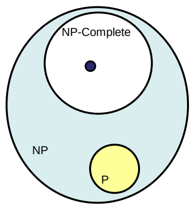
Circuit Satisfiability is NP Complete





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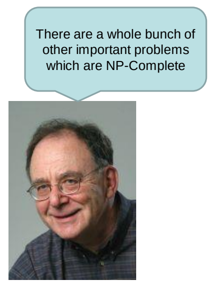
Dick Karp



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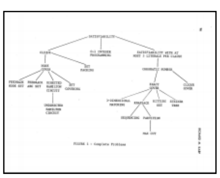
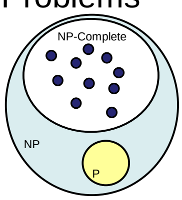
Karp's 21 NP-Complete Problems





- Circuit Sat $<_P$ 3-SAT
- 3-SAT $<_P$ Independent Set
- 3-SAT $<_P$ Vertex Cover
- Independent Set $<_P$ Clique
- 3-SAT $<_P$ Hamiltonian Circuit
- Hamiltonian Circuit $<_P$ Traveling Salesman
- 3-SAT $<_P$ Integer Linear Programming
- 3-SAT $<_P$ Graph Coloring
- 3-SAT $<_P$ Subset Sum
- Subset Sum $<_P$ Scheduling with Release times and deadlines

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