**Announcements**

- Final Exam, March 18, 2:30-4:20 pm
- Practice Exams available

**Today’s topics**

- Network flow reductions
  - Multi source flow
  - Reviewer Assignment
- Baseball Scheduling
- Image Segmentation
- Reading: 7.5, 7.6, 7.10-7.12

**Network Flow Definitions**

- Flowgraph: Directed graph with distinguished vertices s (source) and t (sink)
- Capacities on the edges, c(e) >= 0
- Problem, assign flows f(e) to the edges such that:
  - 0 <= f(e) <= c(e)
  - Flow is conserved at vertices other than s and t
    - Flow conservation: flow going into a vertex equals the flow going out
      - The flow leaving the source is as large as possible

**Key Ideas for Network Flow**

- Residual Graph for a Flow
- Augmenting a flow
- Ford Fulkerson Algorithm
- Max Flow / Min Cut Theorem
- Practical Flow Algorithms
- Modelling problems as Network Flow or Minimum Cut
Multi-source network flow

- Multi-source network flow
  - Sources $s_1, s_2, \ldots, s_k$
  - Sinks $t_1, t_2, \ldots, t_j$
- Solve with Single source network flow

Bipartite Matching

- A graph $G=(V,E)$ is bipartite if the vertices can be partitioned into disjoint sets $X,Y$
- A matching $M$ is a subset of the edges that does not share any vertices
- Find a matching as large as possible

Converting Matching to Network Flow

Integrality Theorem

Theorem: If all capacities are integers, then there exists a maximum flow where all edges are assigned integer valued flows

Resource Allocation: Assignment of reviewers

- A set of papers $P_1, \ldots, P_n$
- A set of reviewers $R_1, \ldots, R_m$
- Paper $P_i$ requires $A_i$ reviewers
- Reviewer $R_j$ can review $B_j$ papers
- For each reviewer $R_j$, there is a list of paper $L_{j1}, \ldots, L_{jk}$ that $R_j$ is qualified to review

Baseball elimination

- Can the Dinosaurs win the league?
- Remaining games:
  - AB, AC, AD, AD, AD, BC, BC, BC, BD, CD

A team wins the league if it has strictly more wins than any other team at the end of the season.
A team ties for first place if no team has more wins, and there is some other team with the same number of wins.
Baseball elimination

• Can the Fruit Flies win or tie the league?
• Remaining games:

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Assume Fruit Flies win remaining games

• Fruit Flies are tied for first place if no team wins more than 19 games
• Allowable wins:
  – Ants (2)
  – Bees (3)
  – Cockroaches (3)
  – Dinosaurs (5)
  – Earthworms (5)
• 18 games to play:

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Remaining games

AC, AD, AD, BC, BC, BC, BD, BE, BE, BE, CE, CE, CE, DE

Image Segmentation

• Separate foreground from background

Minimum Cut Applications

• Image Segmentation
• Open Pit Mining / Task Selection Problem
• Reduction to Min Cut problem

S, T is a cut if S, T is a partition of the vertices with s in S and t in T
The capacity of an S, T cut is the sum of the capacities of all edges going from S to T
Image analysis

- $a_i$: value of assigning pixel $i$ to the foreground
- $b_i$: value of assigning pixel $i$ to the background
- $p_{ij}$: penalty for assigning $i$ to the foreground, $j$ to the background or vice versa
- A: foreground, B: background
- $Q(A,B) = \sum_{i \in A} a_i + \sum_{j \in B} b_j - \sum_{(i,j) \in E, i \in A, j \in B} p_{ij}$

Pixel graph to flow graph

Min cut Construction

[Diagram of pixel graph and flow graph with annotations for $a_i$, $b_i$, $p_{uv}$]