



## CSE 421 Algorithms

Richard Anderson  
Lecture 23  
Network Flow Applications

## Announcements

- Final Exam, March 18, 2:30-4:20 pm
- Practice Exams available

## Today's topics

- Network flow reductions
  - Multi source flow
  - Reviewer Assignment
- Baseball Scheduling
- Image Segmentation
- Reading: 7.5, 7.6, 7.10-7.12

Review

## Network Flow Definitions

- Flowgraph: Directed graph with distinguished vertices  $s$  (source) and  $t$  (sink)
- Capacities on the edges,  $c(e) \geq 0$
- Problem, assign flows  $f(e)$  to the edges such that:
  - $0 \leq f(e) \leq c(e)$
  - Flow is conserved at vertices other than  $s$  and  $t$ 
    - Flow conservation: flow going into a vertex equals the flow going out
  - The flow leaving the source is as large as possible

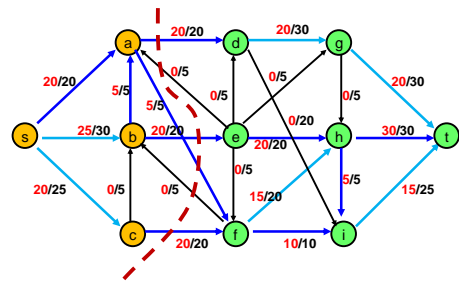
Review

## Key Ideas for Network Flow

- Residual Graph for a Flow
- Augmenting a flow
- Ford Fulkerson Algorithm
- Max Flow / Min Cut Theorem
- Practical Flow Algorithms
- Modelling problems as Network Flow or Minimum Cut

Review

## Max Flow / Min Cut



## Multi-source network flow

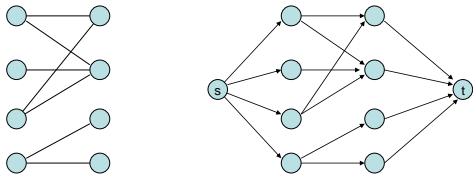
- Multi-source network flow
  - Sources  $s_1, s_2, \dots, s_k$
  - Sinks  $t_1, t_2, \dots, t_j$
- Solve with Single source network flow

Review

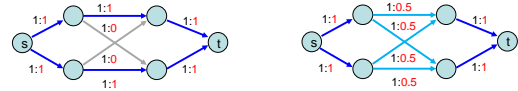
## Bipartite Matching

- A graph  $G=(V,E)$  is bipartite if the vertices can be partitioned into disjoint sets  $X, Y$
- A matching  $M$  is a subset of the edges that does not share any vertices
- Find a matching as large as possible

## Converting Matching to Network Flow



## Integrality Theorem



Theorem: If all capacities are integers, then there exists a maximum flow where all edges are assigned integer valued flows

## Resource Allocation: Assignment of reviewers

- A set of papers  $P_1, \dots, P_n$
- A set of reviewers  $R_1, \dots, R_m$
- Paper  $P_i$  requires  $A_i$  reviewers
- Reviewer  $R_j$  can review  $B_j$  papers
- For each reviewer  $R_j$ , there is a list of paper  $L_{j1}, \dots, L_{jk}$  that  $R_j$  is qualified to review

## Baseball elimination

- Can the Dinosaurs win the league?
- Remaining games:
  - AB, AC, AD, AD, AD, BC, BC, BC, BD, CD

	W	L
Ants	4	2
Bees	4	2
Cockroaches	3	3
Dinosaurs	1	5

A team **wins** the league if it has strictly more wins than any other team at the end of the season  
 A team **ties** for first place if no team has more wins, and there is some other team with the same number of wins

## Baseball elimination

- Can the Fruit Flies win or tie the league?
- Remaining games:
  - AC, AD, AD, AD, AF, BC, BC, BC, BC, BC, BD, BE, BE, BE, BE, BF, CE, CE, CE, CF, CF, DE, DF, EF, EF

	W	L
Ants	17	12
Bees	16	7
Cockroaches	16	7
Dinosaurs	14	13
Earthworms	14	10
Fruit Flies	12	15

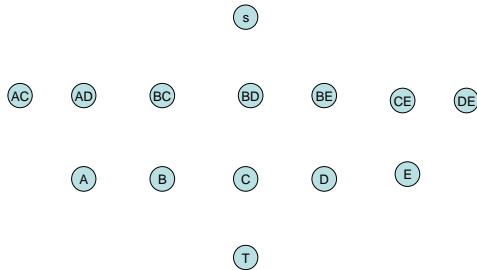
## Assume Fruit Flies win remaining games

- Fruit Flies are tied for first place if no team wins more than 19 games
- Allowable wins
  - Ants (2)
  - Bees (3)
  - Cockroaches (3)
  - Dinosaurs (5)
  - Earthworms (5)
- 18 games to play
  - AC, AD, AD, AD, BC, BC, BC, BC, BC, BD, BE, BE, BE, BE, BE, BE, CE, CE, CE, DE

	W	L
Ants	17	13
Bees	16	8
Cockroaches	16	9
Dinosaurs	14	14
Earthworms	14	12
Fruit Flies	19	15

## Remaining games

AC, AD, AD, AD, BC, BC, BC, BC, BC, BD, BE, BE, BE, BE, CE, CE, CE, DE



## Minimum Cut Applications

- Image Segmentation
- Open Pit Mining / Task Selection Problem
- Reduction to Min Cut problem

$S, T$  is a cut if  $S, T$  is a partition of the vertices with  $s$  in  $S$  and  $t$  in  $T$

The capacity of an  $S, T$  cut is the sum of the capacities of all edges going from  $S$  to  $T$

## Image Segmentation

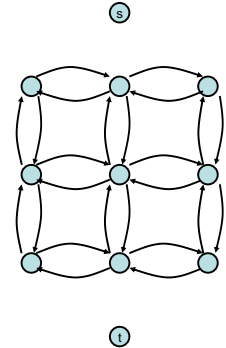
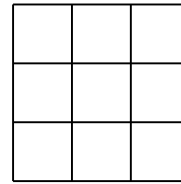
- Separate foreground from background



## Image analysis

- $a_i$ : value of assigning pixel  $i$  to the foreground
- $b_i$ : value of assigning pixel  $i$  to the background
- $p_{ij}$ : penalty for assigning  $i$  to the foreground,  $j$  to the background or vice versa
- $A$ : foreground,  $B$ : background
- $Q(A,B) = \sum_{\{i \text{ in } A\}} a_i + \sum_{\{j \text{ in } B\}} b_j - \sum_{\{(i,j) \text{ in } E, i \text{ in } A, j \text{ in } B\}} p_{ij}$

## Pixel graph to flow graph



## Mincut Construction

