Announcements

- No office hour, Wednesday, Feb 6
- Midterm, Wednesday, Feb 13
  - Coverage through KT 5.2
  - Old midterms posted
- Homework 5, available

**Divide and Conquer**

- Recurrences, Sections 5.1 and 5.2
- Algorithms
  - Fast Matrix Multiplication
  - Counting Inversions (5.3)
  - Closest Pair (5.4)
  - Multiplication (5.5)

```java
// Array Mergesort(Array a){
    n = a.Length;
    if (n <= 1)
        return a;
    b = Mergesort(a[0 .. n/2]);
    c = Mergesort(a[n/2+1 .. n-1]);
    return Merge(b, c);
}
```

**Algorithm Analysis**

- Cost of Merge
- Cost of Mergesort

\[ T(n) = 2T(n/2) + cn; \ T(1) = c; \]
Recurrence Analysis

- Solution methods
  - Unrolling recurrence
  - Guess and verify
  - Plugging in to a "Master Theorem"

Unrolling the recurrence

Substitution

Prove $T(n) \leq cn (\log_2 n + 1)$ for $n \geq 1$

Induction:
Base Case:

Induction Hypothesis:

A better mergesort (?)

- Divide into 3 subarrays and recursively sort
- Apply 3-way merge

What is the recurrence?

Unroll recurrence for $T(n) = 3T(n/3) + dn$

$T(n) = aT(n/b) + f(n)$
Recurrences

- Three basic behaviors
  - Dominated by initial case
  - Dominated by base case
  - All cases equal – we care about the depth