CSE 421
Algorithms

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Lecture 7

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Announcements

- Reading
  - For today, sections 4.1, 4.2, 4.4
  - For friday, sections 4.5, 4.7, 4.8
- Homework 3 is available
  - Random Interval Graphs

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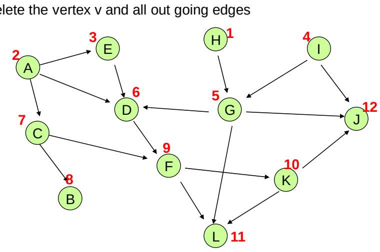
Highlight from last lecture: Topological Sort Algorithm

While there exists a vertex v with in-degree 0

Output vertex v

Delete the vertex v and all out going edges



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Greedy Algorithms

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Greedy Algorithms

- Solve problems with the simplest possible algorithm
- The hard part: showing that something simple actually works
- Pseudo-definition
  - An algorithm is Greedy if it builds its solution by adding elements one at a time using a simple rule

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Scheduling Theory

- Tasks
  - Processing requirements, release times, deadlines
- Processors
- Precedence constraints
- Objective function
  - Jobs scheduled, lateness, total execution time

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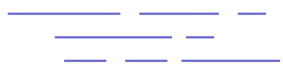
Interval Scheduling

- Tasks occur at fixed times
- Single processor
- Maximize number of tasks completed



- Tasks {1, 2, . . . N}
- Start and finish times, s(i), f(i)

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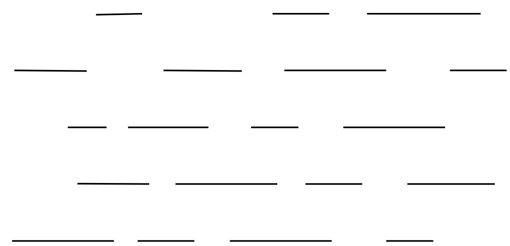
What is the largest solution?



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Greedy Algorithm for Scheduling

Let T be the set of tasks, construct a set of independent tasks I, A is the rule determining the greedy algorithm

I = { }
While (T is not empty)
    Select a task t from T by a rule A
    Add t to I
    Remove t and all tasks incompatible with t from T

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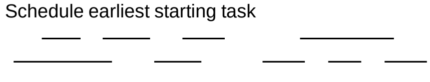
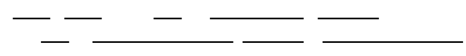
Simulate the greedy algorithm for each of these heuristics

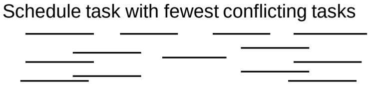
Schedule earliest starting task



Schedule shortest available task



Schedule task with fewest conflicting tasks



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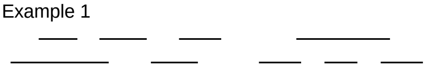
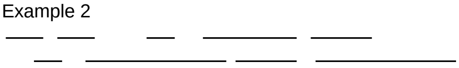
Greedy solution based on earliest finishing time

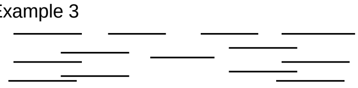
Example 1



Example 2



Example 3



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Theorem: Earliest Finish Algorithm is Optimal

- Key idea: Earliest Finish Algorithm stays ahead
- Let A = $\{i_1, . . ., i_k\}$ be the set of tasks found by EFA in increasing order of finish times
- Let B = $\{j_1, . . ., j_m\}$ be the set of tasks found by a different algorithm in increasing order of finish times
- Show that for r<= min(k, m), $f(i_r) <= f(j_r)$

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Stay ahead lemma

- A always stays ahead of B, $f(i_r) <= f(j_r)$
- Induction argument
  - $f(i_1) <= f(j_1)$
  - If $f(i_{r-1}) <= f(j_{r-1})$ then $f(i_r) <= f(j_r)$

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Completing the proof

- Let A = $\{i_1, . . ., i_k\}$ be the set of tasks found by EFA in increasing order of finish times
- Let O = $\{j_1, . . ., j_m\}$ be the set of tasks found by an optimal algorithm in increasing order of finish times
- If k < m, then the Earliest Finish Algorithm stopped before it ran out of tasks

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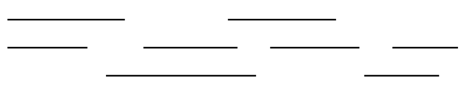
Scheduling all intervals

- Minimize number of processors to schedule all intervals



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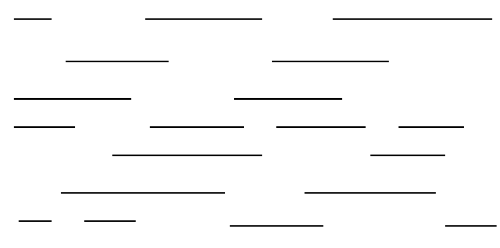
How many processors are needed for this example?



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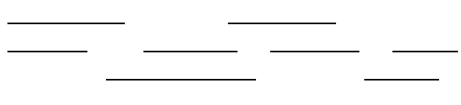
Prove that you cannot schedule this set of intervals with two processors



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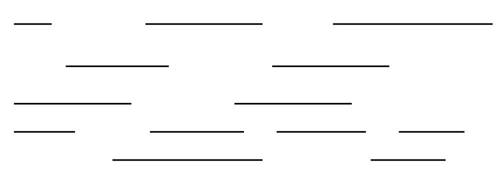
Depth: maximum number of intervals active



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Algorithm

- Sort by start times
- Suppose maximum depth is d, create d slots
- Schedule items in increasing order, assign each item to an open slot

- Correctness proof: When we reach an item, we always have an open slot

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What happens on “Random” sets of intervals

- Given n random intervals
  - What is the expected number independent intervals
  - What is the expected depth

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What is a random set of intervals

- Method 1:
  - Each interval assigned random start position in [0, 1]
  - Each interval assigned a random end length in [0,1]
- Method 2:
  - Start with the array [1, 1, 2, 2, 3, 3, 4, 4, 5, 5]
  - Randomly permute it [2, 1, 4, 2, 3, 4, 5, 1, 3, 5]
  - Index of the first j is the start of interval j, and the index of the second j is the end of interval j

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Scheduling tasks

- Each task has a length t_i and a deadline d_i
- All tasks are available at the start
- One task may be worked on at a time
- All tasks must be completed

- Goal minimize maximum lateness
  - Lateness = $f_i – d_i$ if $f_i >= d_i$

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Example

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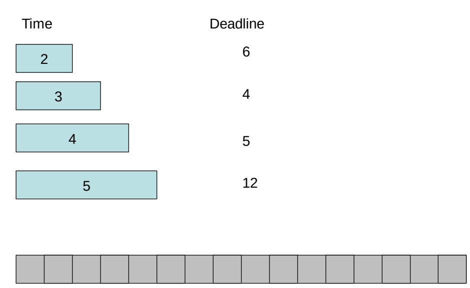
Determine the minimum lateness



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