CSE 421
Algorithms

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Autumn 2019

Lecture 6

Announcements

- No class Monday
  - Richard Anderson will shift office hours to Tuesday, 3:30-4:30 pm, Jan 22 (CSE 582)
- Reading
  - Start on Chapter 4

Stable Matching Results

- Averages of 5 runs
- Much better for M than W
- Why is it better for M?

- What is the growth of m-rank and w-rank as a function of n?

n	m-rank	w-rank
500	5.10	98.05
500	7.52	66.95
500	8.57	58.18
500	6.32	75.87
500	5.25	90.73
500	6.55	77.95
1000	6.80	146.93
1000	6.50	154.71
1000	7.14	133.53
1000	7.44	128.96
1000	7.36	137.85
1000	7.04	140.40
2000	7.83	257.79
2000	7.50	263.78
2000	11.42	175.17
2000	7.16	274.76
2000	7.54	261.60
2000	8.29	246.62

Graph Theory

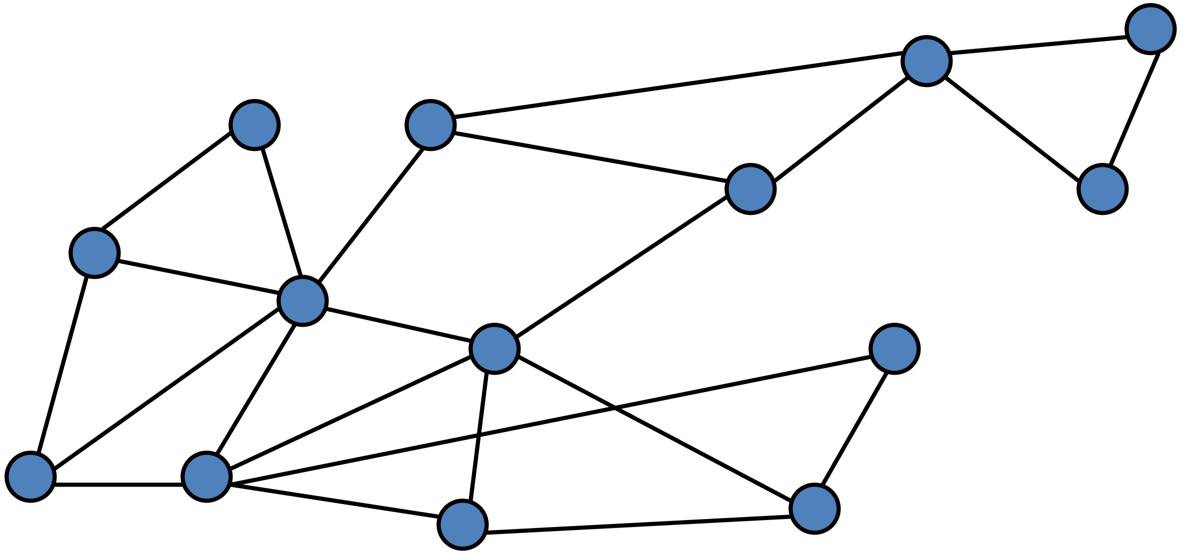
- $G = (V, E)$
  - V: vertices, $|V| = n$
  - E: edges, $|E| = m$
- Undirected graphs
  - Edges sets of two vertices $\{u, v\}$
- Directed graphs
  - Edges ordered pairs (u, v)
- Many other flavors
  - Edge / vertices weights
  - Parallel edges
  - Self loops
- Path: $v_1, v_2, \ldots, v_k$, with (v_i, v_{i+1}) in E
  - Simple Path
  - Cycle
  - Simple Cycle
- Neighborhood
  - $N(v)$
- Distance
- Connectivity
  - Undirected
  - Directed (strong connectivity)
- Trees
  - Rooted
  - Unrooted

Last Lecture

- Bipartite Graphs : two-colorable graphs
- Breadth First Search algorithm for testing two-colorability
  - Two-colorable iff no odd length cycle
  - BFS has cross edge iff graph has odd cycle

Graph Search

- Data structure for next vertex to visit determines search order



Graph search

Breadth First Search

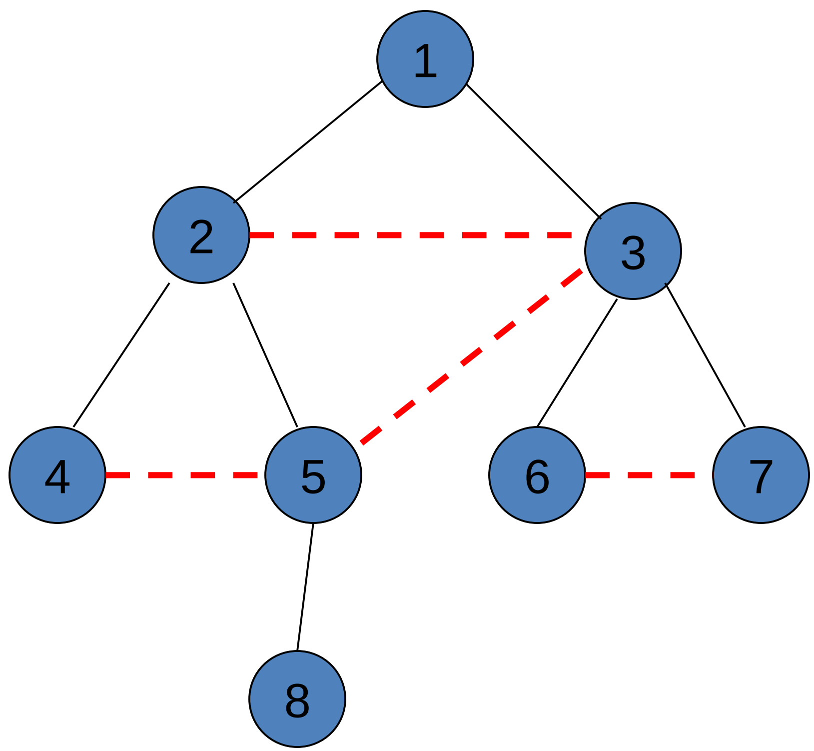
```
S = {s}
while S is not empty
    u = Dequeue(S)
    if u is unvisited
        visit u
        foreach v in N(u)
            Enqueue(S, v)
```

Depth First Search

```
S = {s}
while S is not empty
    u = Pop(S)
    if u is unvisited
        visit u
        foreach v in N(u)
            Push(S, v)
```

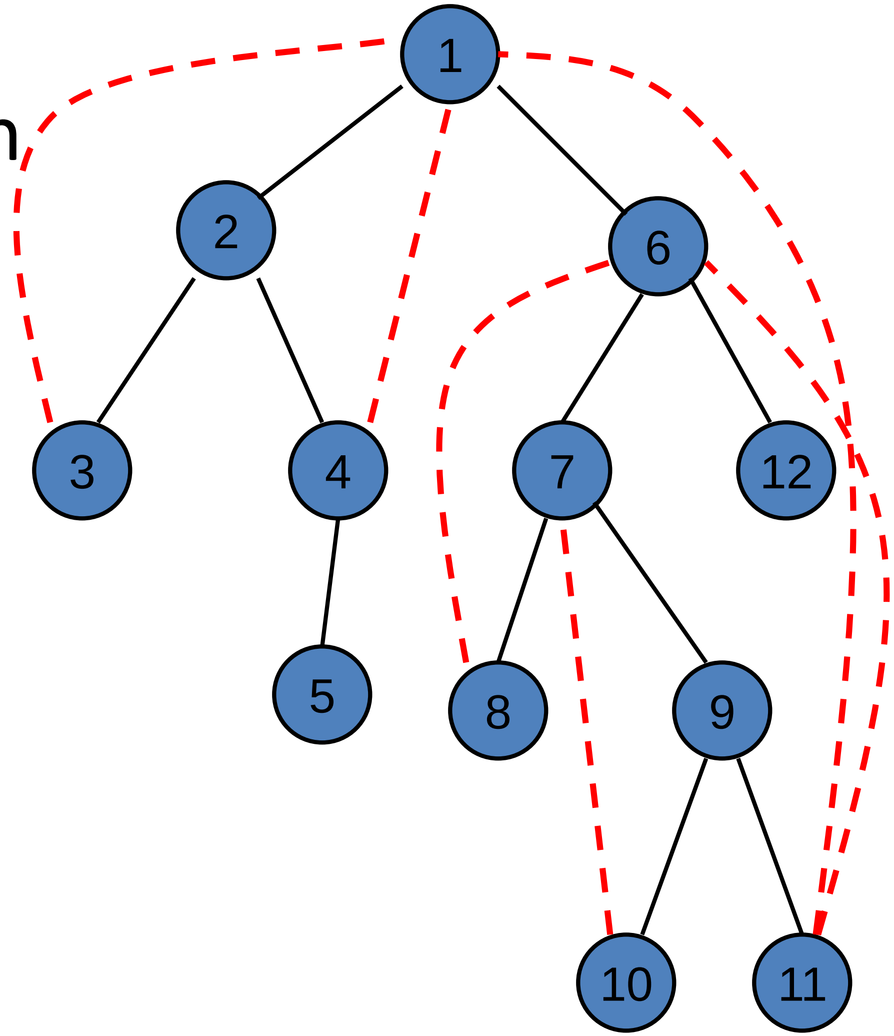
Breadth First Search

- All edges go between vertices on the same layer or adjacent layers



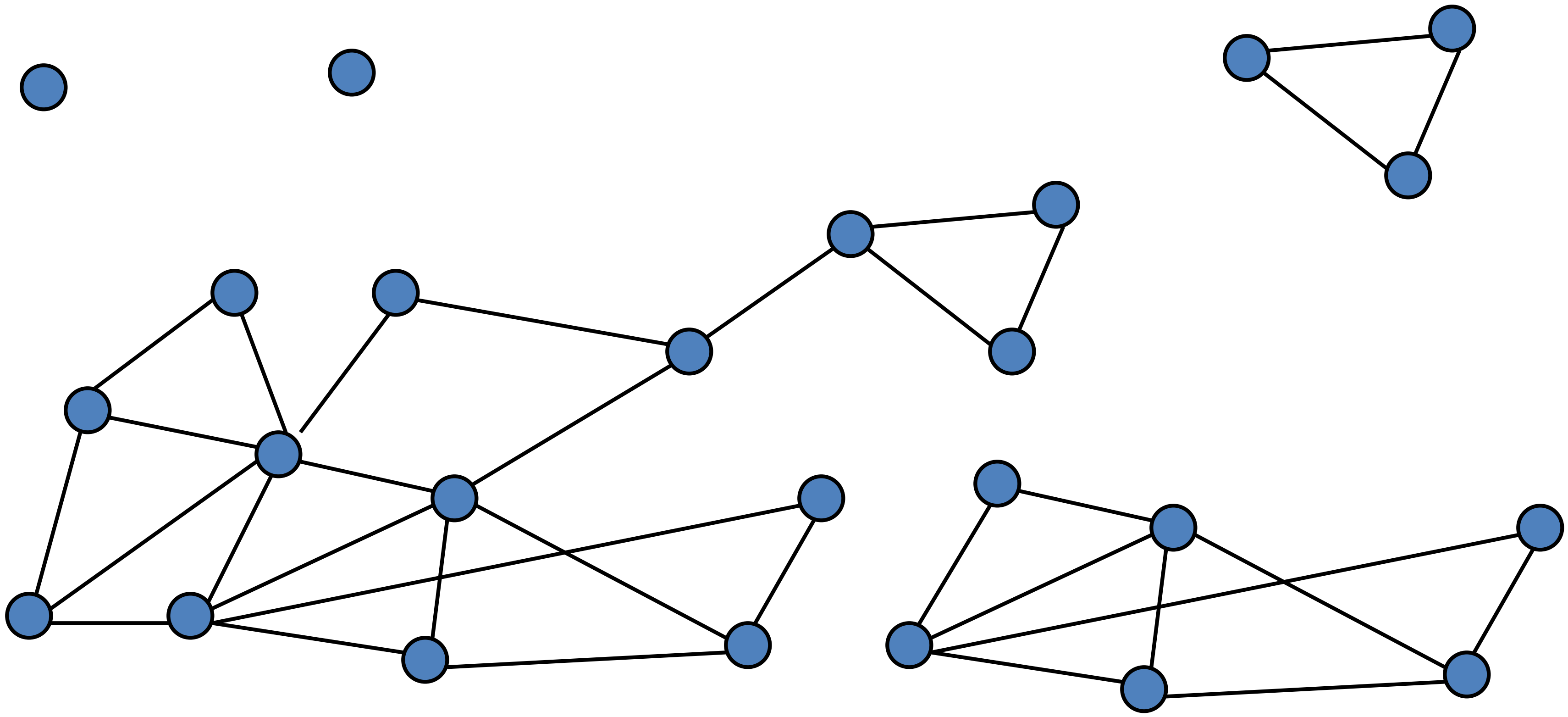
Depth First Search

- Each edge goes between vertices on the same branch
- No cross edges



Connected Components

- Undirected Graphs

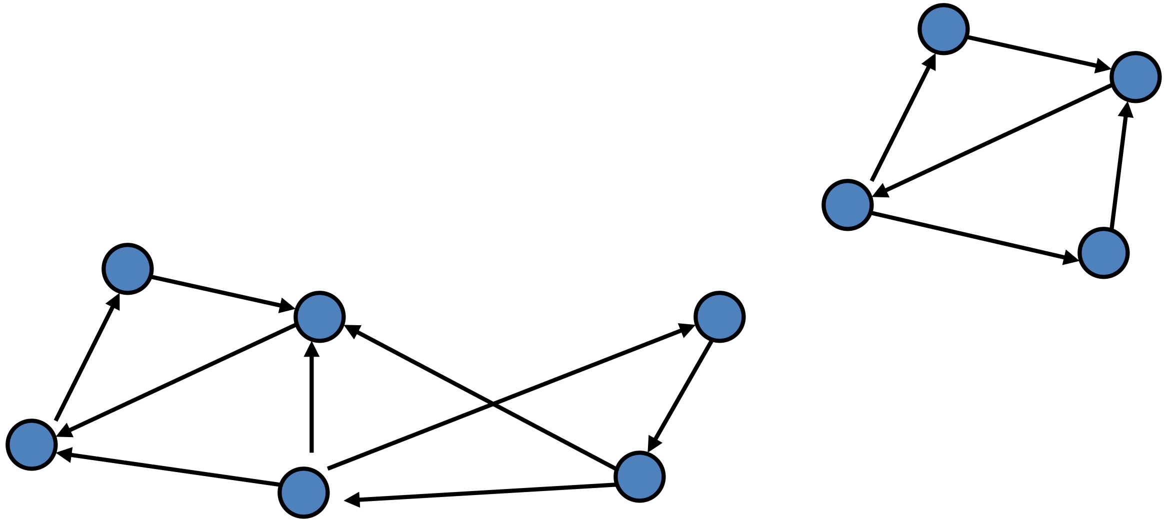


Computing Connected Components in O(n+m) time

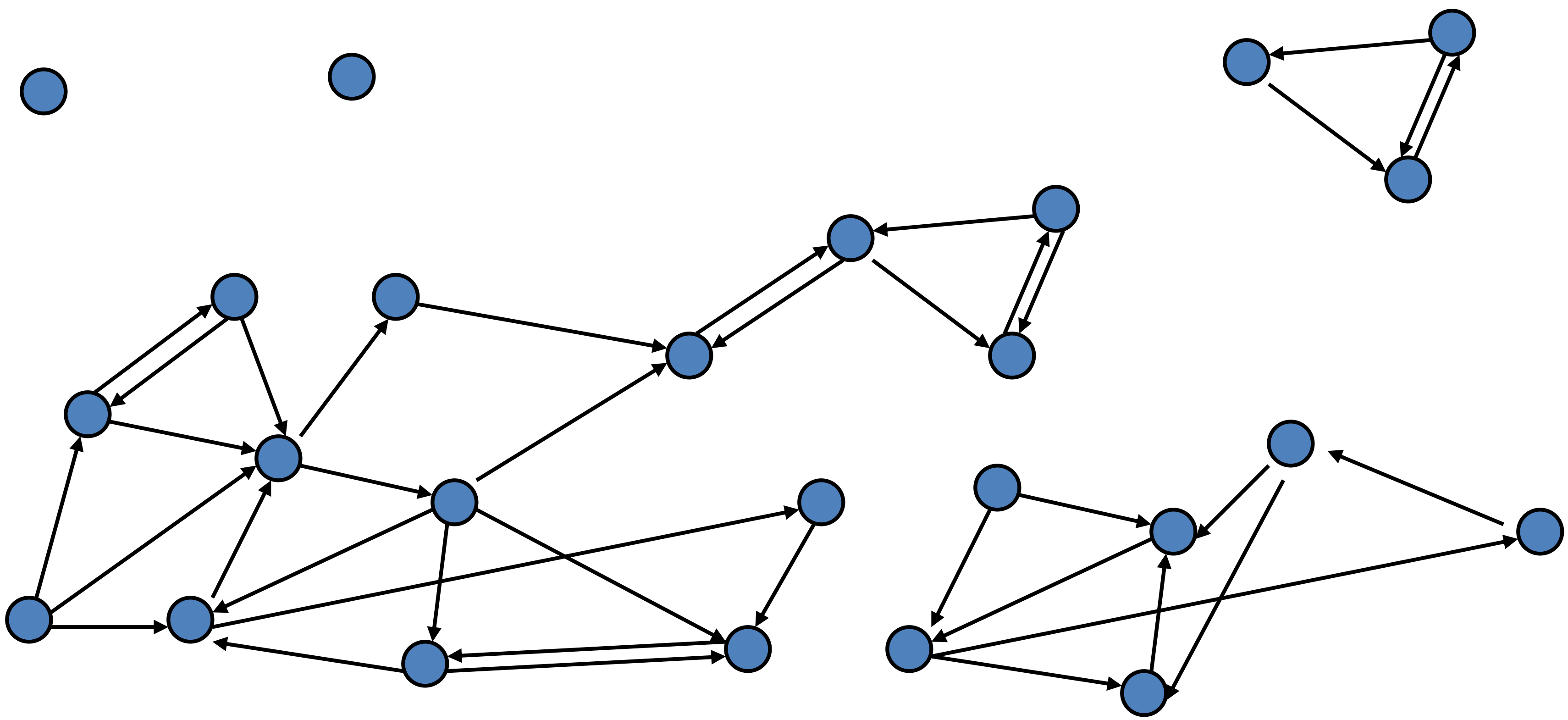
- A search algorithm from a vertex v can find all vertices in v's component
- While there is an unvisited vertex v, search from v to find a new component

Directed Graphs

- A Strongly Connected Component is a subset of the vertices with paths between every pair of vertices.



Identify the Strongly Connected Components

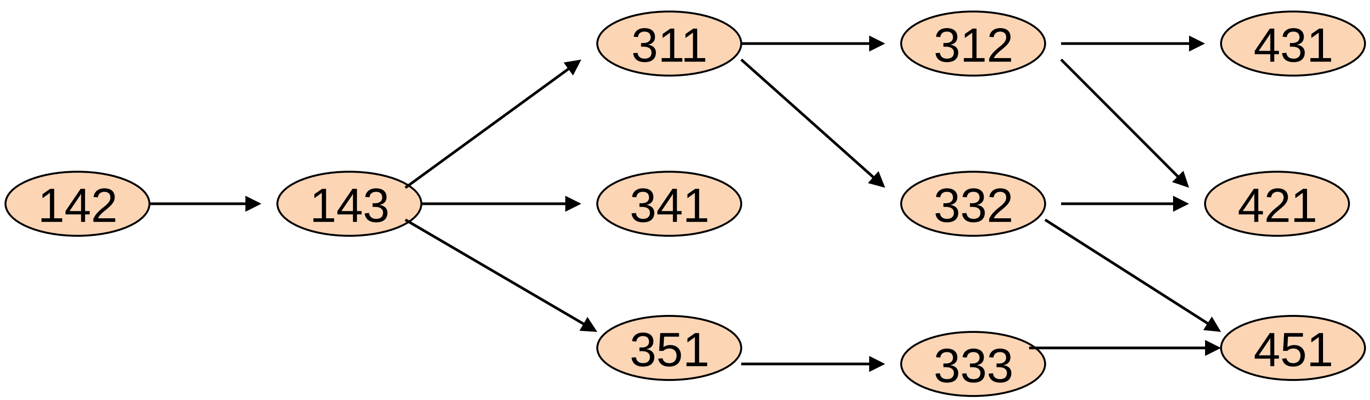


Strongly connected components can be found in O(n+m) time

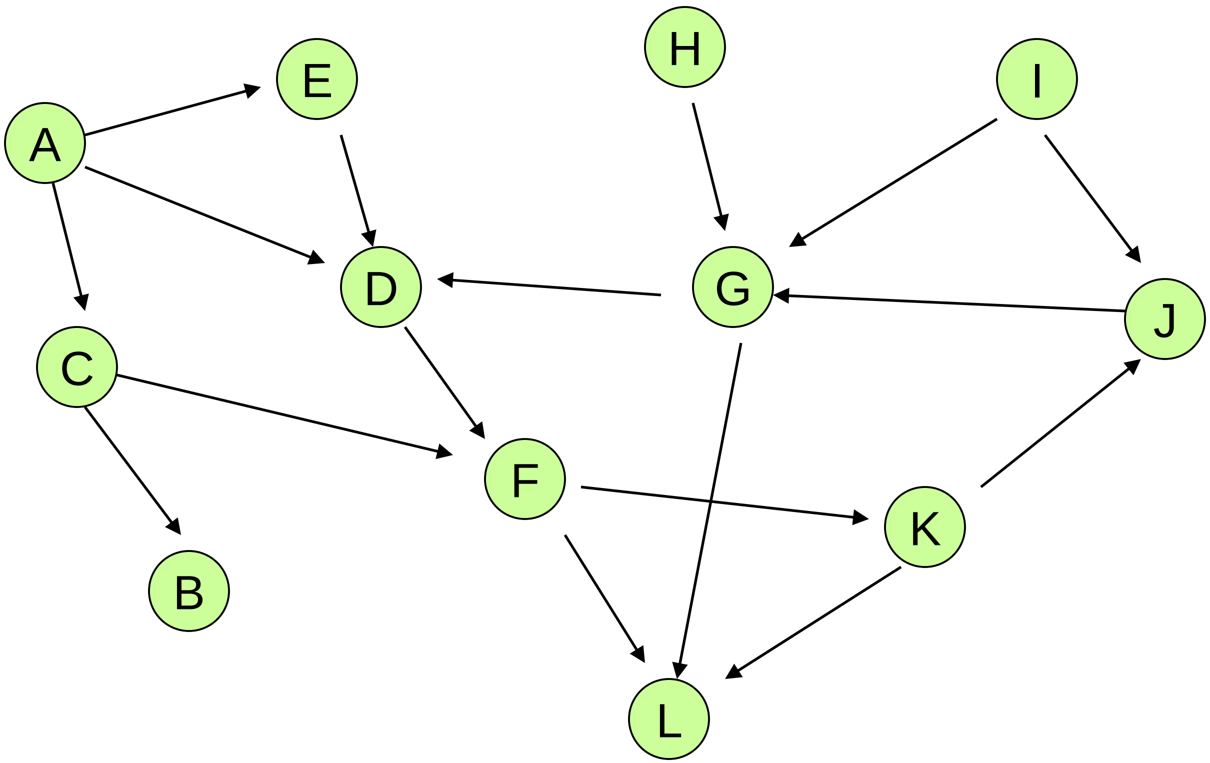
- But it’s tricky!
- Simpler problem: given a vertex v, compute the vertices in v’s scc in O(n+m) time

Topological Sort

- Given a set of tasks with precedence constraints, find a linear order of the tasks

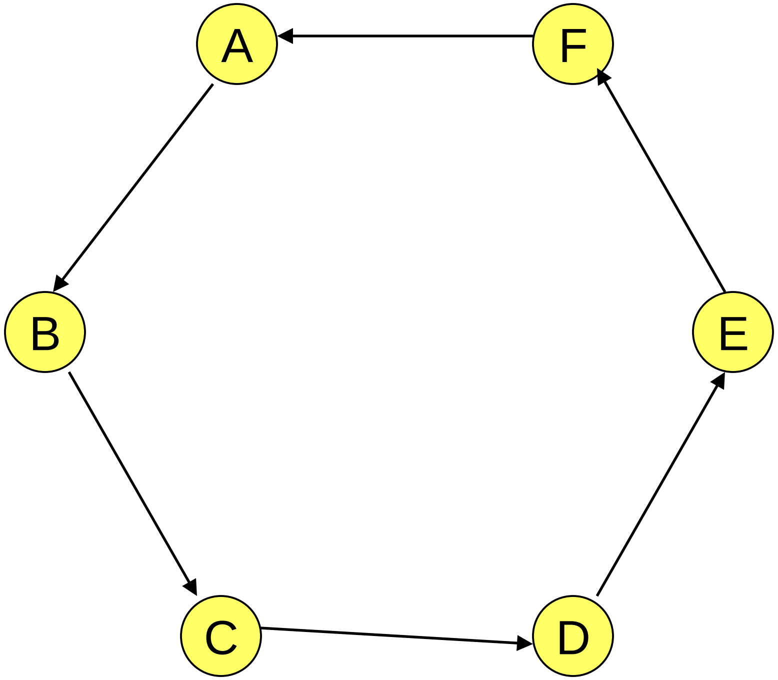


Find a topological order for the following graph



If a graph has a cycle, there is no topological sort

- Consider the first vertex on the cycle in the topological sort
- It must have an incoming edge



Definition: A graph is Acyclic if it has no cycles

Lemma: If a (finite) graph is acyclic, it has a vertex with in-degree 0

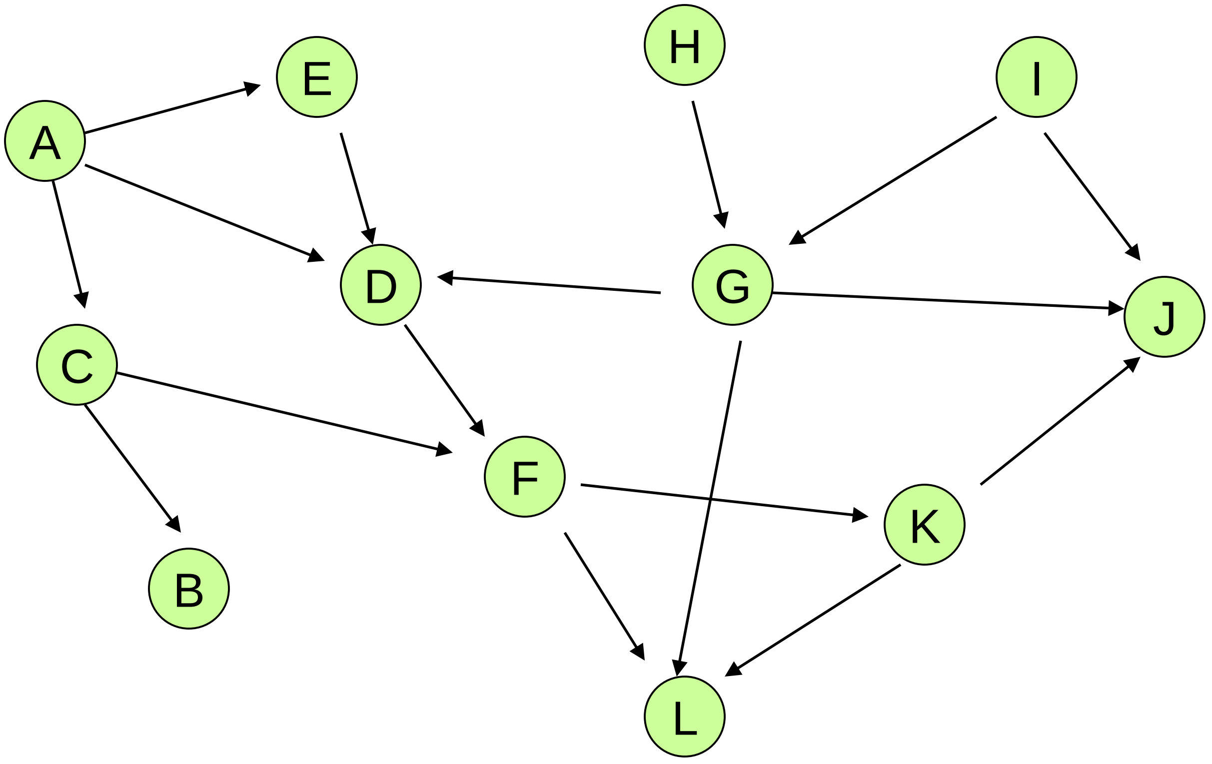
- Proof:
  - Pick a vertex v_1, if it has in-degree 0 then done
  - If not, let (v_2, v_1) be an edge, if v_2 has in-degree 0 then done
  - If not, let (v_3, v_2) be an edge . . .
  - If this process continues for more than n steps, we have a repeated vertex, so we have a cycle

Topological Sort Algorithm

While there exists a vertex v with in-degree 0

Output vertex v

Delete the vertex v and all out going edges



Details for O(n+m) implementation

- Maintain a list of vertices of in-degree 0
- Each vertex keeps track of its in-degree
- Update in-degrees and list when edges are removed
- m edge removals at O(1) cost each