

The slide features five small images: a Gantt chart for a job scheduling problem, a grid with red and blue circles representing a graph or matrix, a network of nodes and edges, a circular graph with nodes and edges, and a grid with red and blue circles representing a graph or matrix.

## Five Problems

CSE 421  
Richard Anderson  
Autumn 2019, Lecture 3

## Announcements

- Course website: [//courses.cs.washington.edu/courses/cse421/19wi/](https://courses.cs.washington.edu/courses/cse421/19wi/)

Lecture Schedule, CSE 421, Winter 2019

Lecture	Date	Topic	Location	Notes
Lecture 1	Monday, 1/14	Course Introduction, Self-Intro	Engineering Center 11	Public Notes: <a href="#">2019-01-14-001</a>
Lecture 2	Wednesday, 1/16	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-01-16-001</a>
Lecture 3	Friday, 1/18	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-01-18-001</a>
Lecture 4	Monday, 1/21	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-01-21-001</a>
Lecture 5	Wednesday, 1/23	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-01-23-001</a>
Lecture 6	Friday, 1/25	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-01-25-001</a>
Lecture 7	Monday, 1/28	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-01-28-001</a>
Lecture 8	Wednesday, 1/30	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-01-30-001</a>
Lecture 9	Friday, 2/1	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-02-01-001</a>
Lecture 10	Monday, 2/4	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-02-04-001</a>
Lecture 11	Wednesday, 2/6	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-02-06-001</a>
Lecture 12	Friday, 2/8	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-02-08-001</a>
Lecture 13	Monday, 2/11	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-02-11-001</a>
Lecture 14	Wednesday, 2/13	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-02-13-001</a>
Lecture 15	Friday, 2/15	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-02-15-001</a>
Lecture 16	Monday, 2/18	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-02-18-001</a>
Lecture 17	Wednesday, 2/20	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-02-20-001</a>
Lecture 18	Friday, 2/22	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-02-22-001</a>
Lecture 19	Monday, 2/25	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-02-25-001</a>
Lecture 20	Wednesday, 2/27	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-02-27-001</a>
Lecture 21	Friday, 2/29	Graphs	Engineering Center 11	Public Notes: <a href="#">2019-02-29-001</a>

- Office hours
  - Richard Anderson
    - Monday, 2:40 pm - 3:30 pm, CSE 582
    - Wednesday, 2:40 pm - 3:30 pm, CSE 582
  - Leiyl Zhang, Monday, 10:30 am-11:20 am, CSE 4th floor breakout
  - Aditya Saraf, Monday, 11:30 am – 12:20 pm, Gates 151
  - Sean Jaffe, Monday, 3:00 pm – 3:50 pm, Gates 152
  - Mathew Luo, Tuesday, 10:30 am – 11:20 am, Gates 151
  - Faye Yu, Tuesday, 2:00 pm – 2:50 pm, CSE 007
  - Anny Kong, Thursday, 1:00 pm – 1:50 pm, CSE 021
  - Phillip Quinn, Friday, 2:30 pm – 3:20 pm, Gates 153
  - Xin Yang, Friday, 4:00 pm – 4:50 pm, CSE 220

## Theory of Algorithms

- What is expertise?
- How do experts differ from novices?

## Introduction of five problems

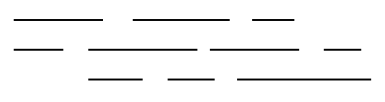
- Show the types of problems we will be considering in the class
- Examples of important types of problems
- Similar looking problems with very different characteristics
- Problems
  - Scheduling
  - Weighted Scheduling
  - Bipartite Matching
  - Maximum Independent Set
  - Competitive Facility Location

## What is a problem?

- Instance
- Solution
- Constraints on solution
- Measure of value

## Problem: Scheduling

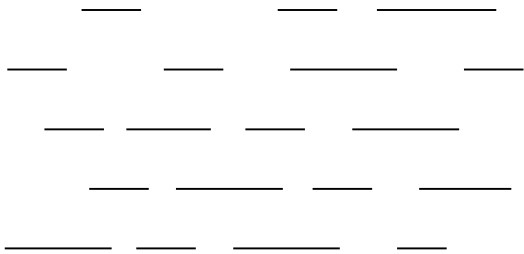
- Suppose that you own a banquet hall
- You have a series of requests for use of the hall:  $(s_1, f_1), (s_2, f_2), \dots$



The diagram shows a horizontal line representing the banquet hall. Below it, several horizontal bars of varying lengths and positions represent requests for use of the hall. Some bars overlap, while others do not.

- Find a set of requests as large as possible with no overlap

### What is the largest solution?

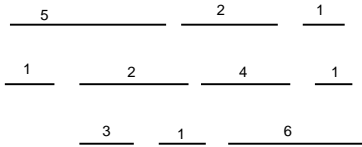


### Greedy Algorithm

- Test elements one at a time if they can be members of the solution
- If an element is not ruled out by earlier choices, add it to the solution
- Many possible choices for ordering (length, start time, end time)
- For this problem, considering the jobs by increasing end time works

### Suppose we add values?

- $(s_i, f_i, v_i)$ , start time, finish time, payment
- Maximize value of elements in the solution



### Greedy Algorithms

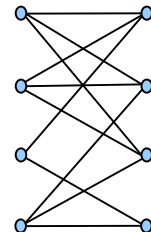
- Earliest finish time
- Maximum value
- Give counter examples to show these algorithms don't find the maximum value solution

### Dynamic Programming

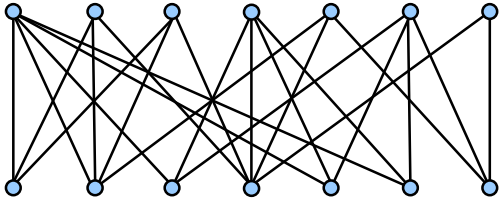
- Requests  $R_1, R_2, R_3, \dots$
- Assume requests are in increasing order of finish time ( $f_1 < f_2 < f_3 \dots$ )
- $Opt_i$  is the maximum value solution of  $\{R_1, R_2, \dots, R_i\}$  containing  $R_i$
- $Opt_i = \text{Max}\{j \mid f_j < s_i\} [Opt_j + v_i]$

### Matching

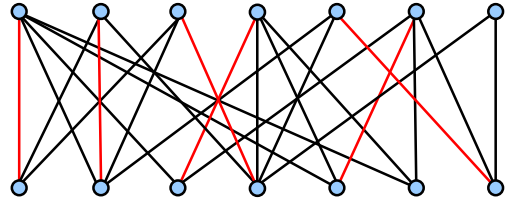
- Given a bipartite graph  $G=(U,V,E)$ , find a subset of the edges  $M$  of maximum size with no common endpoints.
- Application:
  - U: Professors
  - V: Courses
  - $(u,v)$  in E if Prof. u can teach course v



### Find a maximum matching

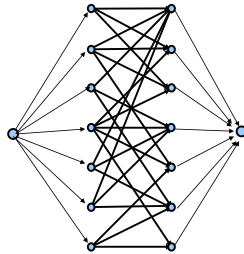


### Augmenting Path Algorithm



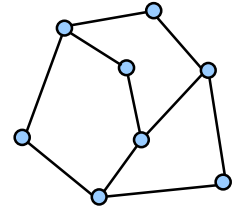
### Reduction to network flow

- More general problem
- Send flow from source to sink
- Flow subject to capacities at edges
- Flow conserved at vertices
- Can solve matching as a flow problem

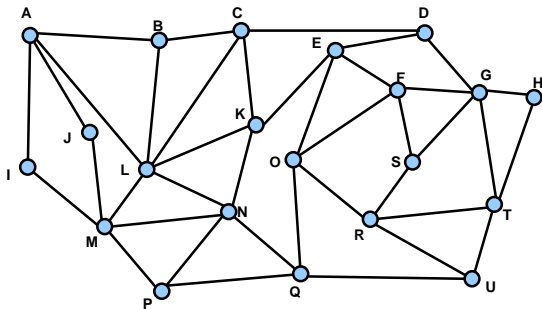


### Maximum Independent Set

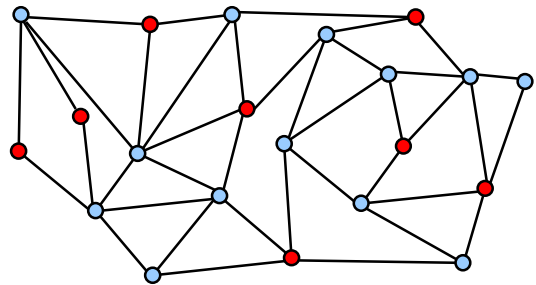
- Given an undirected graph  $G=(V,E)$ , find a set  $I$  of vertices such that there are no edges between vertices of  $I$
- Find a set  $I$  as large as possible



### Find a Maximum Independent Set



### Verification: Prove the graph has an independent set of size 8



## Key characteristic

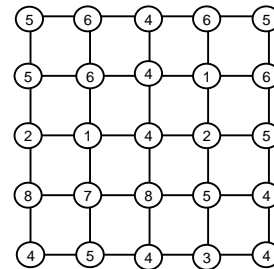
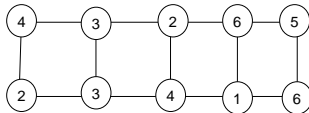
- Hard to find a solution
- Easy to verify a solution once you have one
- Other problems like this
  - Hamiltonian circuit
  - Clique
  - Subset sum
  - Graph coloring

## NP-Completeness

- Theory of Hard Problems
- A large number of problems are known to be equivalent
- Very elegant theory

## Are there even harder problems?

- Simple game:
  - Players alternating selecting nodes in a graph
    - Score points associated with node
    - Remove nodes neighbors
  - When neither can move, player with most points wins



## Competitive Facility Location

- Choose location for a facility
  - Value associated with placement
  - Restriction on placing facilities too close together
- Competitive
  - Different companies place facilities
    - E.g., KFC and McDonald's

## Complexity theory

- These problems are P-Space complete instead of NP-Complete
  - Appear to be much harder
  - No obvious certificate
    - G has a Maximum Independent Set of size 10
    - Player 1 wins by at least 10 points

## Summary

- Scheduling
- Weighted Scheduling
- Bipartite Matching
- Maximum Independent Set
- Competitive Scheduling