University of Washington
Department of Computer Science and Engineering
CSE 421, Winter 2019

January 30, 2019

# Homework 4, Due Wednesday, February 6, 1:29 pm, 2019

Turnin instructions: Electronics submission on canvas using the CSE 421 canvas site. Each numbered problem is to be turned in as a separate PDF.

**Problem 1 (10 points):**

Let $S$ be a set of intervals, where $S = \{I_1, \ldots, I_n\}$ with $I_j = (s_j, f_j)$ and $s_j < f_j$. A set of points $P = \{p_1, \ldots, p_k\}$ is said to be a *cover* for $S$ if every interval of $S$ includes at least one point of $P$, or more formally: for every $I_i$ in $S$, there is a $p_j$ in $P$ with $s_i \leq p_j \leq f_i$.

Describe an algorithm that finds a cover for $S$ that is as small as possible, and prove that your algorithm finds a minimum size cover. You algorithm should be efficient. In this case $O(n \log n)$ is achievable.

**Problem 2 (10 points):**

Suppose you are given a connected graph $G$, with edge costs that are all distinct. Prove that $G$ has a unique minimum spanning tree.

**Problem 3 (10 points):**

Let $G = (V, E)$ be a directed acyclic graph with lengths assigned to the edges. Give an $O(n + m)$ time algorithm that given vertices $s, t \in V$ finds a maximum length path from $s$ to $t$. Justify that your algorithm is correct.

**Problem 4 (10 points):**

Let $G = (V, E)$ be a directed graph with lengths assigned to the edges. Let $\delta(u, v)$ denote the shortest path distance from $u$ to $v$. Prove that for all vertices $u, v, w \in V$:

$$\delta(u, w) \leq \delta(u, v) + \delta(v, w).$$

You may assume that the graph is strongly connected, so that there is a path between every pair of vertices.

**Problem 5 (10 points):**

Let $G = (V, E)$ be a connected, undirected graph with weights on the edges. In this problem, the edge costs need not be distinct, so there may be multiple minimum spanning trees. Suppose that $T$ is a spanning tree with the property that every edge $e \in T$ is in *some* minimum spanning tree for $G$. Is $T$ necessarily a minimum spanning tree? Give a proof or a counterexample with an explanation.

**Problem 6 (10 points):**

Let $G = (V, E)$ be a directed graph with integral edge costs in $\{1, 2, 3, 4, 5, 6, 7, 8\}$. Give an $O(n + m)$ time algorithm that given vertices $s, t \in V$ finds a shortest path from $s$ to $t$.