Thm: Greedy is optimal for job scheduling. [schedule max # of compatible jobs].

Pf [Greedy is ahead].

$\underbrace{f(i_1) \leq f(i_2) \leq \cdots f(i_k)}_{\text{greedy}}$ $\qquad$ $\underbrace{f(j_1) \leq f(j_2) \leq \cdots \leq f(j_m)}_{\text{OPT}}$.

Goal: $k \geq m$. $\qquad$ [Careful OPT is not necessarily uniq].

Lem: For all $r \geq 1$, $f(i_r) \leq f(j_r)$.

Pf of Lem. Use induction.

Base Case $r=1$. ✓ First job in greedy has smallest finishing time.

IH. Supp for some $r \geq 1$, $f(i_r) \leq f(j_r)$.

IS. To show $f(i_{r+1}) \leq f(j_{r+1})$.

We know $f(i_r) \leq f(j_r) \leq s(j_{r+1}) \Rightarrow j_{r+1}$ is comp with i_r.

So j_{r+1} is a feasible option for Greedy when scheduling i_{r+1}.

$\Rightarrow f(i_{r+1}) \leq f(j_{r+1})$. $\square$

If $k<m$, j_{k+1} is compatible with i_k BC $f(i_k) \leq f(j_k) \leq s(j_{k+1})$.

So Greedy should schedule j_{k+1}. contradiction!

Thm: Greedy uses minimum # of classes.

Pf. Supp Greedy uses d classes.

Goal $d \overset{?}{\leq} \text{depth} \leq \text{OPT}$

To show, enough to exhibit a vertical line which crosses d classes.

Look at time, t, that greedy allocates d-th classroom.

$d-1$

Direction: Ew ALG uses $\geq$ depth classrooms

At this time we had allocated $d-1$ classrooms, and they were all occupied. A new job arrives at time t, so $t+\varepsilon$ for some small $\varepsilon > 0$ gives a line crossing d open intervals. $\square$