Claim: For evry edge $\{x,y\}$, $|L(x) - L(y)| \leq 1$.

Pf. Suppose x is discovrd first.

We process queue until we get to x. $\textcircled{x}$

Case 1. y is not yet disc when we process x.

$\rightarrow$ y will be disc when process x and we set $L(y) = L(x)+1$.

Case 2. y is disc before we process x,

Say y is disc when we process z, and z is before x

in queue $\underbrace{L(x) \leq}_{x \text{ is add bef } y} \underbrace{L(y) =}_{y \text{ is disc when proc } z.} \underbrace{L(z)+1 \leq}_{z \text{ is add bef } x} L(x)+1$

Claim. For all i, $L(i) =$ shortest path of s to i (when we do BFS(s)).

Pf. - shortest path $\leq L(i)$.

$L(i) =$ length of path from $s \to i$ in BFS tree

$\vee|$

shortest path $=$ length of the smallest path $s \to i$

- $L(i) \leq$ shortest path.

shortest path $s = v_0, v_1 \text{ ---- }, v_k = i$

$L(v_0) = 0$

$L(v_1) \leq L(v_0) + 1 = 1$

$L(v_2) \leq L(v_1) + 1 \leq 2$

$\vdots$

$L(i) = L(v_k) \leq k$.

$\Rightarrow$ shortest path to i $= k \geq L(i) = L(v_k)$

Exercise: If G has n vertices & n edges $\Rightarrow$ G has a cycle.