Claim: In any graph $G=(V,E)$ $\sum_v \deg(v) = 2\cdot|E|$

Pf. Cl. For every graph G with k edges, $\sum \deg_G(v) = 2k$.

Base Case $k=0$. $\sum \deg_G(v) = 0 = 2\cdot 0 = 0$.

IH. For every graph with $k-1$ edges, claim holds.

IS. Supp G has k edges. Remove an edge $\{x,y\}$ and call the new graph G'.

By IH, $\sum_v \deg_{G'}(v) = 2(k-1)$.

$\forall v \neq y,x$: $\deg_{G'}(v) = \deg_G(v)$. $\quad 1+\deg_{G'}(x) = \deg_G(x) \quad 1+\deg_{G'}(y) = \deg_G(y)$

$$\Rightarrow \sum \deg_G(v) = 2 + \sum \deg_{G'}(v) = 2 + 2(k-1) = 2k.$$

Claim: Let G be a graph with no cycles. Then G has a vertex of degree ≤ 1.

Pf. [by contradiction]

Supp $\forall v$ $\deg(v) \geq 2$. $\quad$ Goal: G has a cycle.

$$v_1 - v_2 - v_3 - \cdots \; v_i$$

If v_i is connected to one of $v_1 \ldots v_{i-2}$ $\Rightarrow$ G has a cycle

not possible.

By assumption, $\deg(v_i) \geq 2 \Rightarrow v_i$ has a neighbor other than v_{i-1}. Call that v_{i+1}.

G has finite size $\Rightarrow$ this cannot continue infinitely.

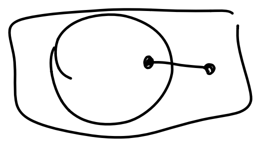
$\Rightarrow$ either we get a cycle or a vertex of $\deg$ 1.

contradiction!

Claim. Show that every tree with n vertices must have $n-1$ edges

Pf.

IS. T with n vertices. Add a new vertex and connect it to one of vertices of T.



T'

T' has $n+1$ vertice and n edges ✓

incorrect! Not clear if you can construct all trees this way.

IS. Start with T with $n+1$ vertices. We know T has a leaf, say x. Remove x from T and call the remaining graph T'.

T' has n vertices

Claim T' is a tree:

- no cycles. We cannot create cycles by removing edges

  T has no cycle $\Rightarrow$ T' has no cycle

- connected: No path in T goes through x (since x is a leaf) T is conn $\Rightarrow$ T' connected.

T' is a tree $\Rightarrow$ by IH T' has $n-1$ edges $\Rightarrow$ T has n edges.

IH: Every tree with n vertices has $n-1$ edges.