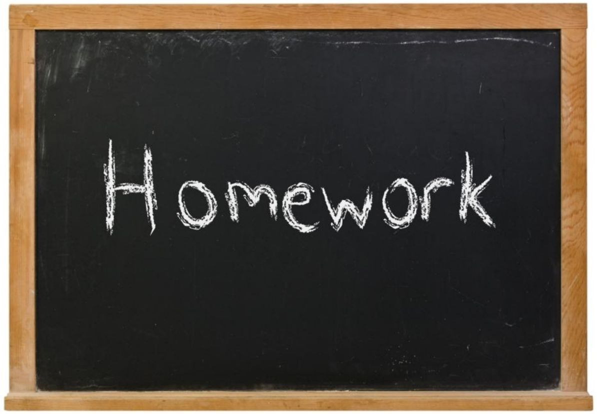
Def of stable match.
M is stable if M has no ~~match~~ unstable pair

# CSE 421

## Course Overview / Complexity

Course Contents

# Administrativia Stuffs




HW1 is out!
It is due Thursday April 11 at 5:00
Please submit to Canvas

Late Submission: Coordinate with me
How to submit?

- Submit a separate file for each problem
- Double check your submission before the deadline!!
- For hand written solutions, take a picture, turn it into pdf and submit

Guidelines:

- Always justify your answer
- You can collaborate, but you must write solutions on your own
- Your proofs should be clear, well-organized, and concise. Spell out main idea.
- Sanity Check: Make sure you use assumptions of the problem

# Extensions: Matching Residents to Hospitals

Men ≈ hospitals, Women ≈ med school residents.

- Variant 1: Some participants declare others as unacceptable.
- Variant 2: Unequal number of men and women.
- Variant 3: Limited polygamy.

e.g. A resident not interested in Cleveland

e.g. A hospital wants to hire **3** residents

Def: Matching **S** is unstable if there is hospital **h** and resident **r** s.t.

- **h** and **r** are acceptable to each other; and
- either **r** is unmatched, or **r** prefers **h** to her assigned hospital; and
- either **h** does not have all its places filled, or **h** prefers **r** to at least one of its assigned residents.

# Lessons Learned

- Powerful ideas learned in course. [give names]
  - Isolate underlying structure of problem.
  - Create useful and efficient algorithms.

- Potentially deep social ramifications. [legal disclaimer]
  - Historically, men propose to women. Why not vice versa?
  - Men: propose early and often.
  - Men: be more honest.
  - Women: ask out the guys.
  - Theory can be socially enriching and fun!

# "The Match": Doctors and Medical Residences


- Each medical school graduate submits a ranked list of hospital where he wants to do a residency

- Each hospital submits a ranked list of newly minted doctors

- A computer runs stable matching algorithm (extended to handle polygamy)

- Until recently, it was hospital-optimal.

# History

1900

- Idea of hospital having residents (then called “interns”)

1900-1940s

- Intense competition among hospitals
  - Each hospital makes offers independently
  - Process degenerates into a race; hospitals advancing date at which they finalize binding contracts

1944

- Medical schools stop releasing info about students before a fixed date

1945-1949

- Hospitals started putting time limits on offers
  - Time limits down to 12 hours; lots of unhappy people

# "The Match"

1950

- NICI run a centralized algorithm for a trial run
- The pairing was not stable, Oops!!

1952

- The algorithm was modified and adopted. It was called the Match.
- The first matching produced in April 1952

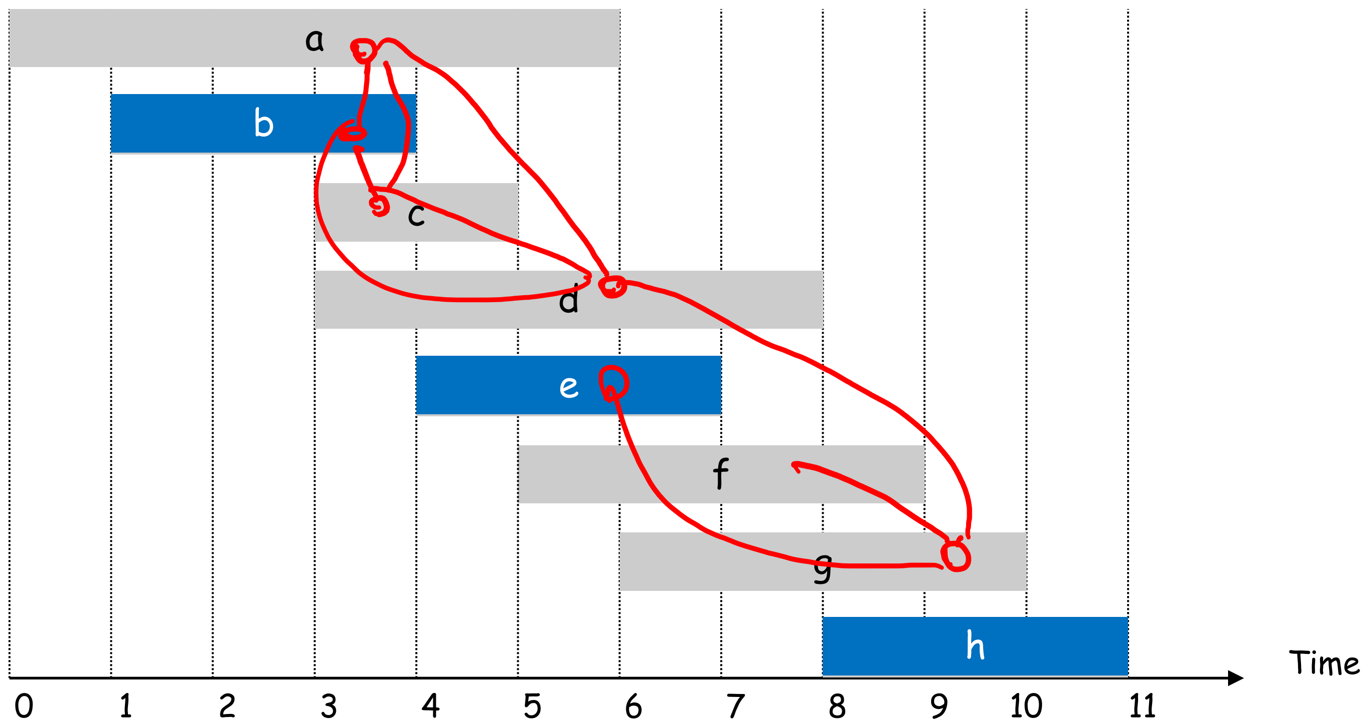
# Five Representative Problems

1. Interval Scheduling
2. Weighted Interval Scheduling
3. Bipartite Matching
4. Independent Set Problem
5. Competitive Facility Location

# Interval Scheduling

Input: Given a set of jobs with start/finish times

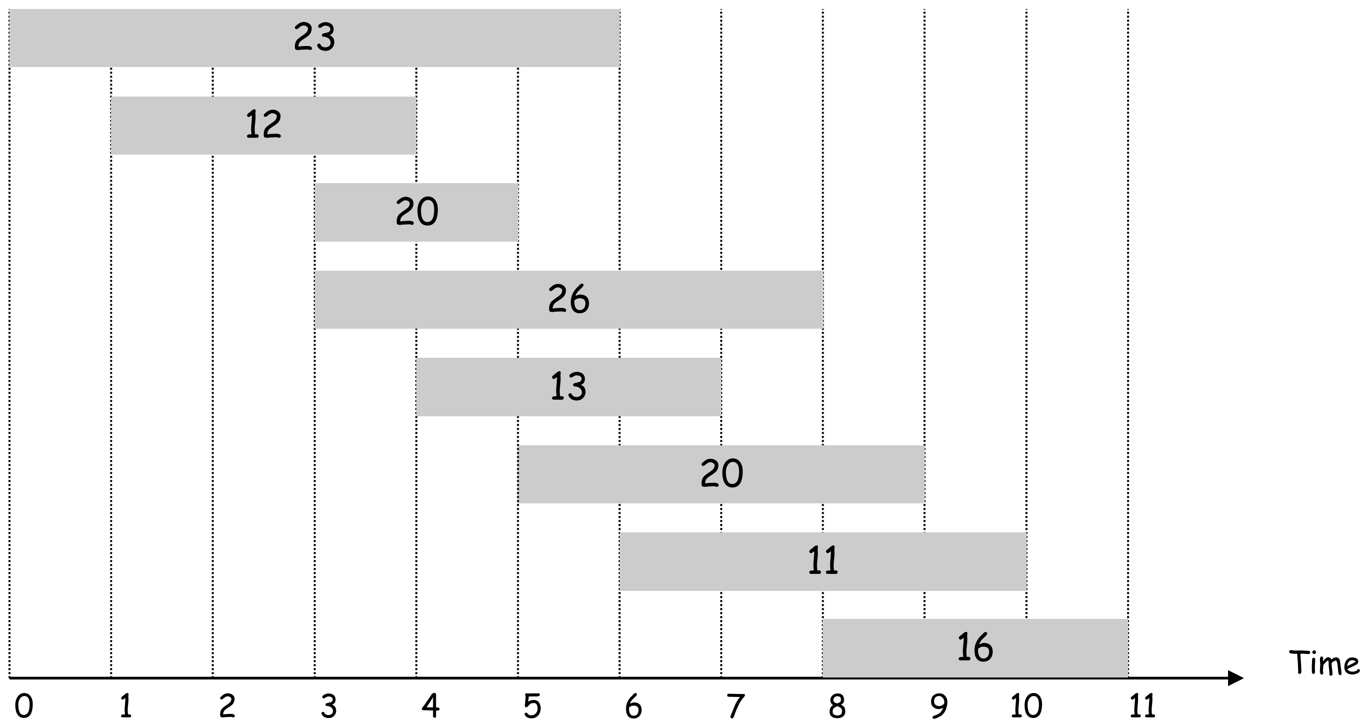
Goal: Find the maximum cardinality subset of jobs that can be run on a single machine.



# Interval Scheduling

Input: Given a set of jobs with start/finish times

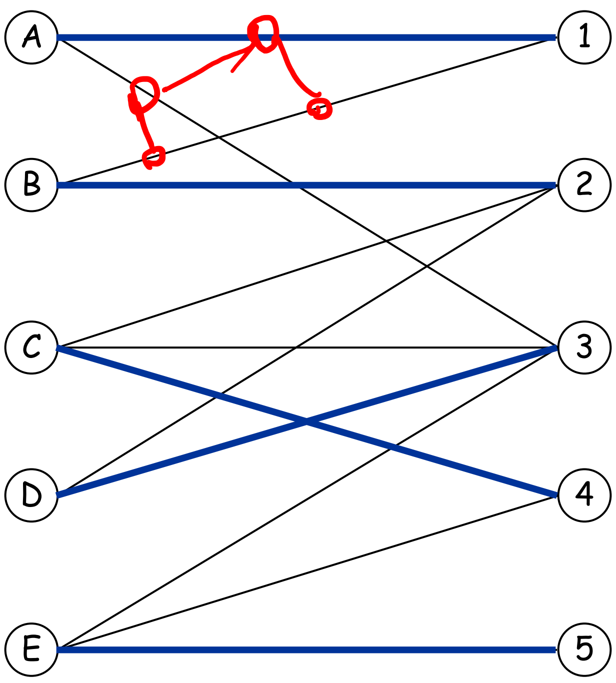
Goal: Find the maximum weight subset of jobs that can be run on a single machine.



# Bipartite Matching

Input: Given a bipartite graph

Goal: Find the maximum cardinality matching

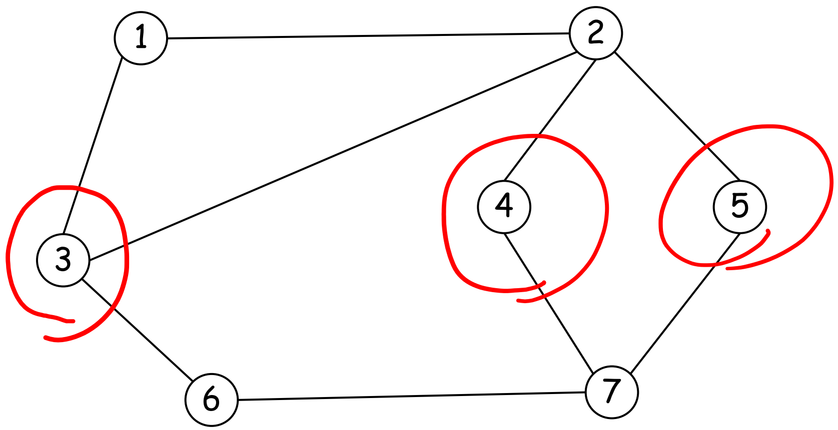


# Independent Set

Input: A graph

Goal: Find the maximum independent set

Subset of nodes that no two joined by an edge

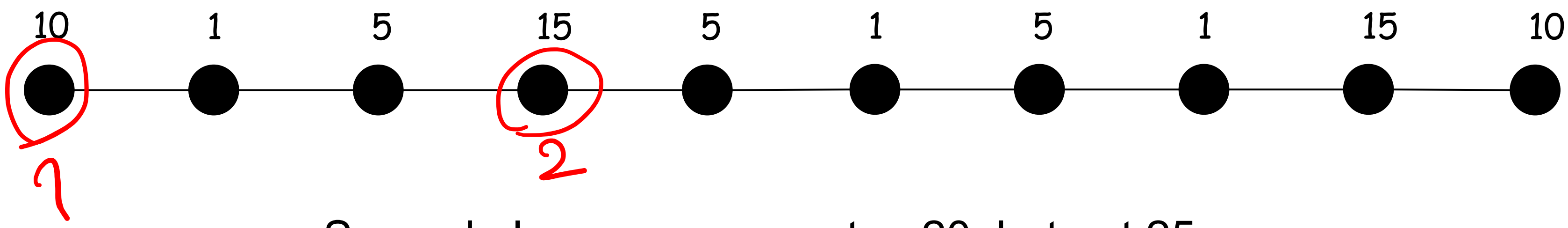


# Competitive Facility Location

Input: Graph with weight on each node

Game: Two competing players alternate in selecting nodes. Not allowed to select a node if any of its neighbors have been selected.

Goal. Does player 2 have a strategy which guarantees a total value of $V$ no matter what player 1 does?




Second player can guarantee 20, but not 25.

# Five Representative Problems

Variation of a theme: Independent set Problem

1. Interval Scheduling
   $n \log n$ greedy algorithm

2. Weighted Interval Scheduling
   $n \log n$ dynamic programming algorithm

3. Bipartite Matching
   $n^k$ maximum flow based algorithm

4. Independent Set Problem: NP-complete

5. Competitive Facility Location: PSPACE-complete

# Defining Efficient Algorithms

# Defining Efficiency

"Runs fast on typical real problem instances"

Pros:

- Sensible,
- Bottom-line oriented

Cons:

- Moving target (diff computers, programming languages)
- Highly subjective (how fast is "fast"? What is "typical"?)

# Measuring Efficiency

Time ≈ # of instructions executed in a simple programming language

- only simple operations (+,*,-,=,if,call,…)
- each operation takes one time step
- each memory access takes one time step
- no fancy stuff (add these two matrices, copy this long string,…) built in; write it/charge for it as above

# Time Complexity

Problem: An algorithm can have different running time on different inputs

Solution: The complexity of an algorithm associates a number **T(N)**, the "time" the algorithm takes on problem size **N**.

On which inputs of size **N**?

Mathematically,

- **T** is a function that maps positive integers giving problem size to positive integers giving number of steps

# Time Complexity (N)

Worst Case Complexity: max # steps algorithm takes on any input of size **N**

This Couse

Average Case Complexity: avg # steps algorithm takes on inputs of size **N**

Best Case Complexity: min # steps algorithm takes on any input of size **N**

# Why Worst-case Inputs?

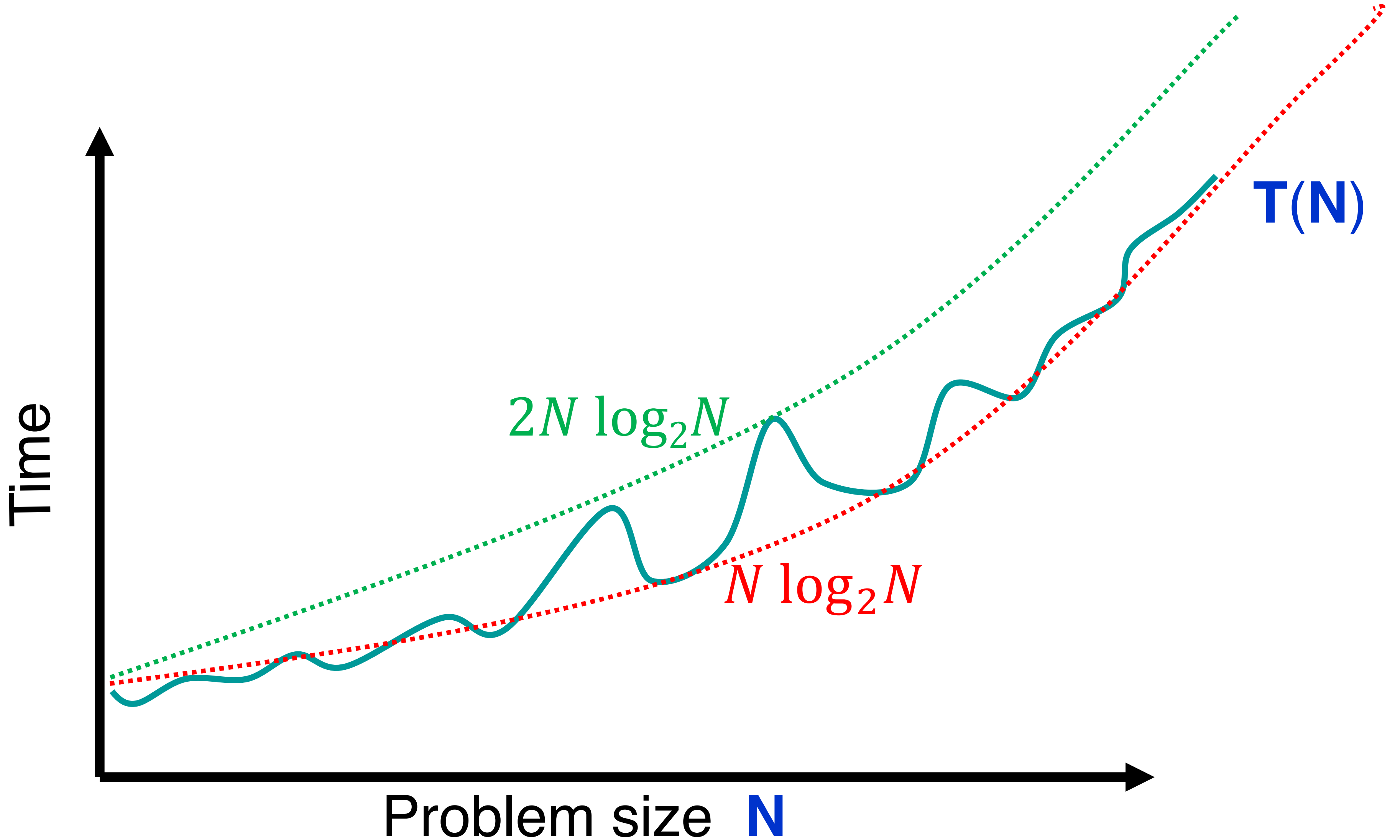
- Analysis is typically easier
- Useful in real-time applications

  e.g., space shuttle, nuclear reactors)
- Worst-case instances kick in when an algorithm is run as a module many times

  e.g., geometry or linear algebra library
- Useful when running competitions

  e.g., airline prices
- Unlike average-case no debate about the right definition

# Time Complexity on Worst Case Inputs



# O-Notation

Given two positive functions **f** and **g**

- **f(N)** is **O(g(N))** iff there is a constant **c**$>$**0** s.t.,
  **f(N)** is eventually always $\leq$ **c g(N)**

- **f(N)** is **Ω(g(N))** iff there is a constant **ε**$>$**0** s.t.,
  **f(N)** is $\geq$ **ε g(N)** for infinitely

- **f(N)** is **Θ(g(N))** iff there are constants $c_1$, $c_2>0$ so that
  eventually always $c_1 g(N) \leq f(N) \leq c_2 g(N)$

# Asymptotic Bounds for common fns

- Polynomials:

  $a_0 + a_1 n + \cdots + a_d n^d$ is $O\left(n^d\right)$

- Logarithms:

  $\log_a n = O(\log_b n)$ for all constants $a, b > 0$

  $lg_a n = \frac{lg_2 n}{lg_a 2}$

- Logarithms: log grows slower than every polynomial

  For all $x > 0$, $\log n = O(n^k)$

  $lg\, n = O(n^{0.000001})$

- $n \log n = O(n^{1.01})$

  $n^{1000000} = O(2^n)$

# Efficient = Polynomial Time

An algorithm runs in polynomial time if $T(n)=O(n^d)$ for some constant d independent of the input size n.

Why Polynomial time?

If problem size grows by at most a constant factor then so does the running time $T(N) = n^k$

- E.g. $\mathbf{T(2N) \leq c(2N)^k \leq 2^k(cN^k)}$
- Polynomial-time is exactly the set of running times that have this property

Typical running times are small degree polynomials, mostly less than $\mathbf{N^3}$, at worst $\mathbf{N^6}$, not $\mathbf{N^{100}}$

$2^n$ $2^{100}$

$2^{200}$ ope

# Why it matters?

- #atoms in universe < $2^{240}$
- Life of the universe < $2^{54}$ seconds
- A CPU does < $2^{30}$ operations a second

$2^{84} \cdot 2^{240}$

If every atom is a CPU, a $2^n$ time ALG cannot solve n=350 if we start at Big-Bang.

| | $n$ | $n \log_2 n$ | $n^2$ | $n^3$ | $1.5^n$ | $2^n$ | $n!$ |
|---|---|---|---|---|---|---|---|
| $n = 10$ | < 1 sec | < 1 sec | < 1 sec | < 1 sec | < 1 sec | < 1 sec | 4 sec |
| $n = 30$ | < 1 sec | < 1 sec | < 1 sec | < 1 sec | < 1 sec | 18 min | $10^{25}$ years |
| $n = 50$ | < 1 sec | < 1 sec | < 1 sec | < 1 sec | 11 min | 36 years | very long |
| $n = 100$ | < 1 sec | < 1 sec | < 1 sec | 1 sec | 12,892 years | $10^{17}$ years | very long |
| $n = 1{,}000$ | < 1 sec | < 1 sec | 1 sec | 18 min | very long | very long | very long |
| $n = 10{,}000$ | < 1 sec | < 1 sec | 2 min | 12 days | very long | very long | very long |
| $n = 100{,}000$ | < 1 sec | 2 sec | 3 hours | 32 years | very long | very long | very long |
| $n = 1{,}000{,}000$ | 1 sec | 20 sec | 12 days | 31,710 years | very long | very long | very long |

not only get very big, but do so *abruptly*, which likely yields errati performance on small instances

# Why "Polynomial"?

Point is not that $n^{2000}$ is a practical bound, or that the differences among n and 2n and $n^2$ are negligible.

Rather, simple theoretical tools may not easily capture such differences, whereas exponentials are qualitatively different from polynomials, so more amenable to theoretical analysis.

- "My problem is in P" is a starting point for a more detailed analysis
- "My problem is not in P" may suggest that you need to shift to a more tractable variant