

1 Asymptotics

Some properties of asymptotics:

- If $f \leq O(g)$ and $g \leq O(h)$ then $f \leq O(h)$.
- If $f \geq \Omega(g)$ and $g \geq \Omega(h)$ then $f \geq \Omega(h)$.
- If $f = \Theta(g)$ and $g = \Theta(h)$ then $f = \Theta(h)$.
- If $f = O(h)$, $g = O(h)$ then $f + g = O(h)$.

Some common running times:

- Polynomial: $O(n^d)$. Exponential $2^{O(n)}$, Logarithmic $O(\log n)$.
- For every positive ϵ (no matter how small), $\log n \leq O(n^\epsilon)$. For every positive d (no matter how large), $n^d \leq O(2^n)$.

2 In class exercise

Arrange in increasing order of asymptotic growth. All logs are in base 2.

- $n^{5/3}$
- $2^{\sqrt{\log n}}$
- $\sqrt{n^n}$
- $\frac{n^2}{\log n}$
- 2^n .

Hint:

- Always keep in mind $n = 2^{\log_2 n}$. For example, $n^{1.5} = 2^{1.5 \log_2 n}$.
- Also recall $2^a \cdot 2^b = 2^{a+b}$ and that $(2^a)^b = 2^{a \cdot b}$