Claim: $G$ has indep set of size $\geq k \iff G'$ has a clique of size $\geq k$.

$\Rightarrow$

$G$ has indep set $S$ s.t. $|S| \geq k$.
So $S$ is a clique in $G'$
So $G'$ has a clique of size $\geq k$.

$\Leftarrow$

$G'$ has a clique $S$ s.t. $|S| \geq k$.
So $S$ is an indep set in $C$. 

Proof of correctness:

Bipars max match $\leq_P$ General graph max matching

max ind on trees $\leq_P$ max ind set on graphs
So $G$ has an independent set of size $7k$. 

* $S$ is a vertex cover if for every edge $e$ 1 endpoint is in $S$.
* $S$ is an independent set if for any edge 1 endpoint in $S$.

* e.g. 

$S_a = \{1, 4\}$  $S_b = \{1, 2, 3\}$  $S_c = \{2, 5\}$  $S_d = \{3, 4, 5\}$.

$S_b$ and $S_d$ cover $\{1, 5\}$.

$G$ 

$U \subseteq E \rightarrow \text{yes}$
Correctness: Suppose Vertex Cov \((G, k) = \text{yes}\). \(S\) is a vertex cover of \(G\) of size \(\leq k\).

For all \(v \in S\), add \(S_v\).

Because \(S\) is a vertex, it covers all edges. So \(S_{\cup V}\) covers all elements of the vertex cover instance as well.

\(\implies\) Suppose Set Cov \((E, \{S_v\}_{v \in V}) = \text{yes}\).

meaning that \(S_v, \ldots, S_v\) cover all elements \(1 \leq i \leq k\).

Then \(v_1, \ldots, v_k\) cover all edges of \(G\).