Find $(A, B)$ max

$$\max \sum_{i \in A} a_i + \sum_{j \in B} b_j - \sum_{\substack{i \in A \\ j \in B \\ \{i,j\} \in E}} p_{i,j}$$

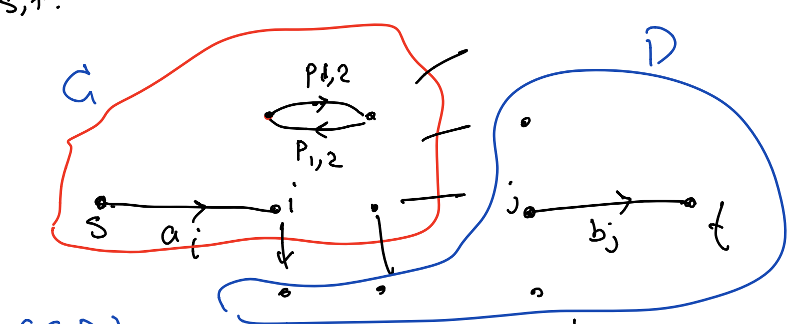
$$\min -\left( \quad \right) = -\sum_{i \in A} a_i - \sum_{j \in B} b_j + \sum_{\substack{i \in A \\ j \in B \\ \{i,j\} \in E}} p_{i,j}$$

Fact: adding a constant to objective fn does not change the optimum solution

add $\sum_i a_i + \sum_j b_j$ (Does not depend on A or B)

Get $\min \sum_{i \in B} a_i + \sum_{j \in A} b_j + \sum_{\substack{i \in A \\ j \in B \\ \{i,j\} \in E}} p_{i,j}$.

add s,t.




$(C, D)$ is a s-t cut: $\underbrace{\sum_{j \in D} a_j}_{\text{edges } s \to D} + \underbrace{\sum_{i \in C} b_i}_{\text{edges } C \to t} + \sum_{\substack{i \in D \\ j \in C \\ \{i,j\} \in E}} p_{i,j}$

Claim: Min s-t cut is OPT image segmentation.

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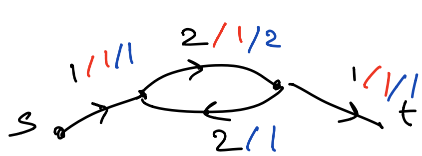

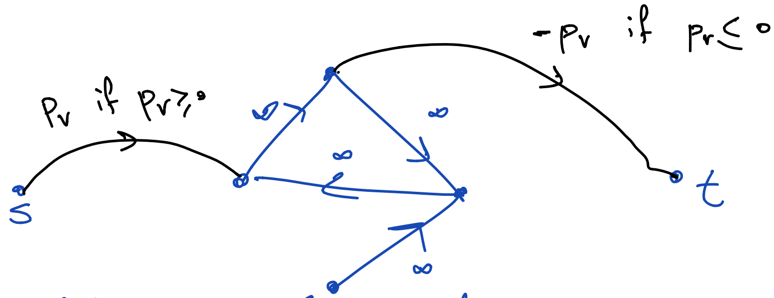


In
Min s–t cut (A,B) no ∞ edge leaves A.
So, all pre-req will be satisfied

If A is my proj $\Rightarrow$ Gain $\sum_{v \in A} p_v$.

Equi: find A (feasible) minimize $\sum_{v \in A} -p_v + \sum_{v : p_v > 0} p_v$

Look at a (s,t) cut (A,B).

$$cap(A,B) = \underbrace{\sum_{\substack{v \in B \\ p_v > 0}} p_v}_{s \rightsquigarrow B} + \sum_{\substack{v \in A \\ p_v < 0}} -p_v$$