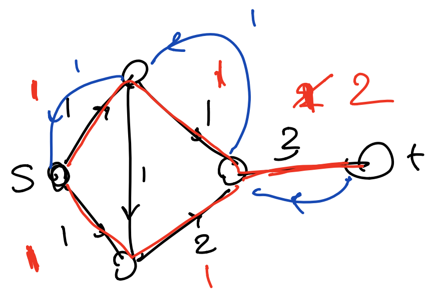
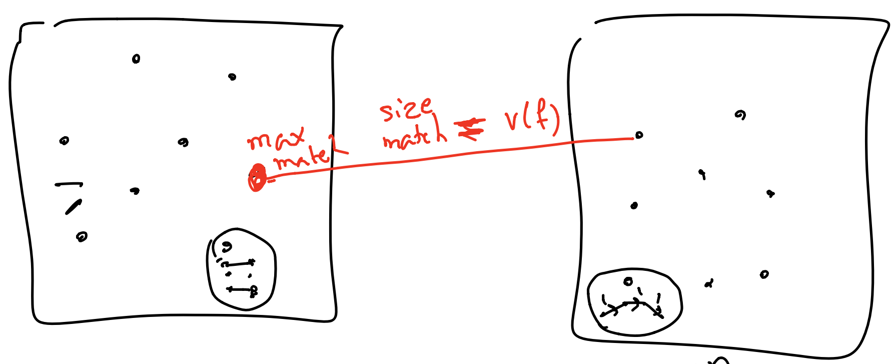
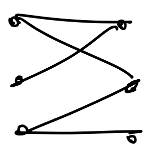
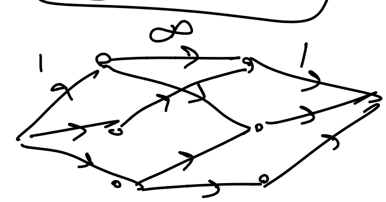






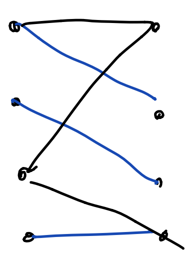


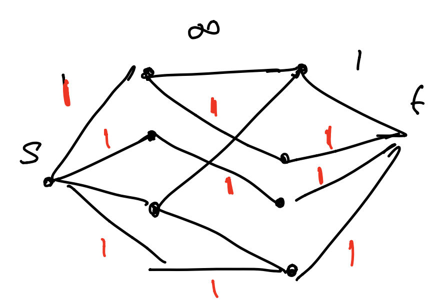
→ 1) max match ≤ max flow

2) max match ≥ max flow

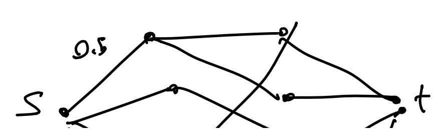
Given M

proves (1)

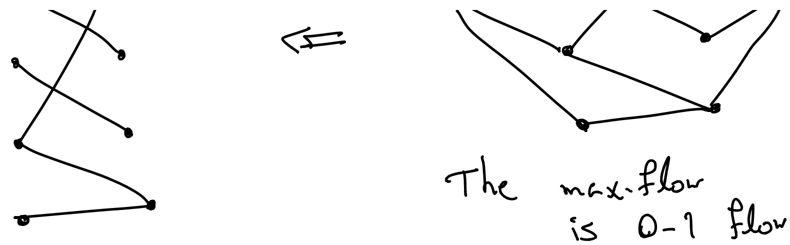




matching?



proves (2)



The max-flow is 0-1 flow.

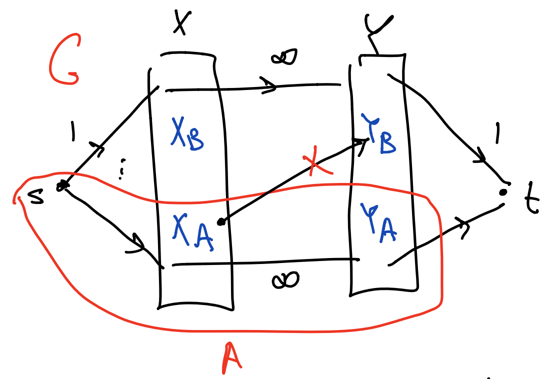
Idea. Include all middle edges of $f(e)=1$ in the matching.

Claim. This gives a matching.

Pf. If not, two edges incident to the same node (say on the left). But then that node has outgoing flow 2, and incoming ≤ 1. Contradiction!

Assume, $\forall S \subseteq X$, $|N(S)| \geq |S| \Rightarrow G$ has a perfect matching.

Pf. By contradiction.



f is max flow. So, $v(f) < |X|$.

(A,B) is min s-t cut.

$$cap(A,B) = v(f) < |X|.$$

$X_A = X \cap A$, $Y_A = Y \cap A$, $X_B = X \cap B$.

Claim: No edge from $X_A \to Y_B$. O/w $cap(A,B) = \infty$ contradiction BC all middle edges have infinite capacity $\square$

$$Cap(A,B) = |X_B| + |Y_A|.$$

Claim. $N(X_A) \subseteq Y_A \Rightarrow |N(X_A)| \leq |Y_A|$.

$$|N(X_A)| \leq |Y_A| = Cap(A,B) - |X_B| < |X| - |X_B| = |X_A|$$

assumption $v(f) < |X|$.

Contradiction!