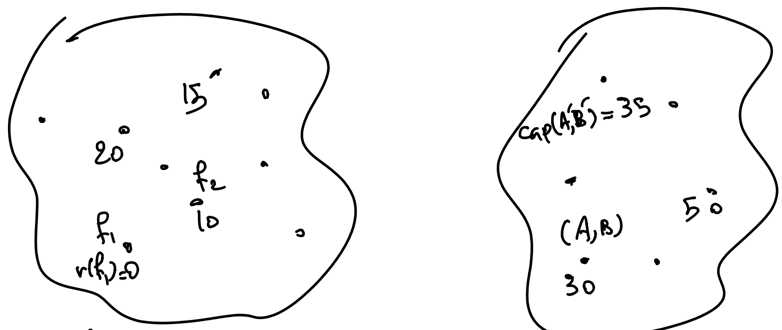
G



flows s-t cuts

$$\forall f, \ \forall A, B$$
$$v(f) \leq cap(A,B).$$

$$\Rightarrow \max_f v(f) \leq cap(A,B) \quad \forall A, B$$

$$\Rightarrow \max_f v(f) \leq \min_{A,B} cap(A,B).$$

$\Rightarrow$ max flow is at most the min-cut. $(\forall G)$.

If $\exists A, B$ such that $cap(A,B) = v(f)$

$\Rightarrow$ f is max-flow and A, B is mincut.

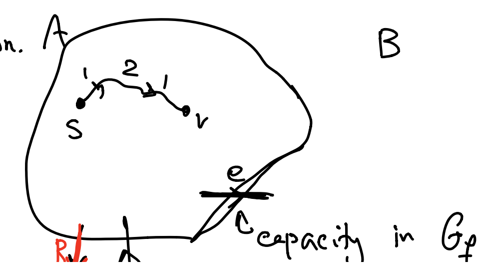
(iii) $\rightarrow$ (i): If no more augmentation $\Rightarrow \exists (A,B)$ s.t.

$v(f) = cap(A,B)$.

Pf. Define A to be all vertices reachable from s by edges of positive capacity in G_f

First $s \in A$. ✓

$t \notin A$. (O/w we have an augmentation.



(A,B) is a s-t cut.

If $e \in G \Rightarrow f(e) = c(e)$.

If a goes to A

(**) g^R goes out of A $\quad f(g) = 0 \qquad g \ldots g \ldots$ is 0.

$$\mathrm{Cap}(A,B) = \sum_{e \text{ out of } A} c(e) \overset{(*)}{=} \sum_{e \text{ out of } A} f(e)$$

$$\overset{(**)}{=} \sum_{e \text{ out of } A} f(e) - \sum_{g \text{ into } A} f(g)$$

$$= v(f)$$