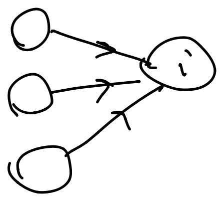
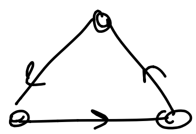
Directed G.

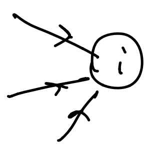
$OPT(i)$ = length of longest path ending at i.



$$OPT(i) = \max_{j:\, j \to i} OPT(j) + 1.$$



$OPT(i, \ell)$ = says is there a path of length ℓ ending at i.



$OPT(i, \ell) =$ for all $j : j \to i$
if $OPT(j, \ell-1) =$ true.
then $OPT(i, \ell) =$ true
and false o/w.

wrong BC I could have visited i to get to j

$OPT(i, S)$: longest path ending at i using vertices in a set S of vertices.

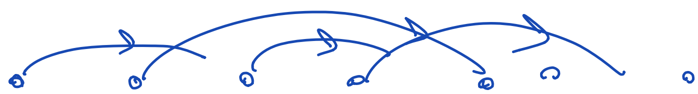
exponentially many states.

Now assume DAG.

OBS: if $j \to i$, i cannot be in the longest path ending at j. O/w we get a cycle.

$OPT(i) = \max^{\text{len}}$ path ending at i.

$$OPT(i) = \max_{j:\, j \to i} OPT(j) + 1.$$



What order? The topological order.

To make sure we compute $OPT(j)$ before $OPT(i)$.

Alternatively $OPT(i)$

$OPT(i) = 0.$

for $j = 1 \ldots n$

if $j \to i$

then $OPT(i) = \max(OPT(i), OPT(j)+1)$

$a_1 \quad a_2 \quad - - - \quad a_n$

if $a_i < a_j$

Note i) This a DAG

ii) Every path corresponds to an increasing subseq

So longest path = longest increasing subseq.

Shortest path with neg edge weights

$OPT(i)$ = shortest path ending at i.

$$OPT(i) = \min_{j:\, j \to i} OPT(j) + c_{j,i}.$$

Problem: What is the right order? Is j closer to s or i is closer?

$OPT(i, \ell)$ = shortest path ending at i which has $\leq \ell$ edges.

$$OPT(i, \ell) = \min_{j : j \to i} OPT(j, \ell - 1) + c_{j,i}.$$