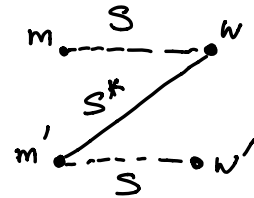


Claim. Output of GS (S^*) is man optimal.

Pf. [by contradiction]. There exist a man who does not receive his best valid partner. ^{OBS!} Man m is the first man who gets rejected by his best valid partner (w).

w rejects m because e. of m' : $\boxed{m' >_w m}$



w is a valid partner of $m \Rightarrow \exists$ stable match S s.t.

$$(m, w) \in S$$

Supp w' is match of m' in S .

If $\boxed{w >_{m'} w'}$ $\Rightarrow (m', w)$ is unstable for S cannot happen X

$\Rightarrow w' >_w w \Rightarrow w'$ has rejected m' .

m' got rejected by a valid partner before m . m was first such man \Rightarrow contradiction.

Claim. GS is woman-pessimal (S^*).

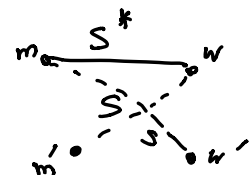
Pf. Suppose $(m, w) \in S^*$, and m is not worst valid partner of w .

\Rightarrow exist a stable matching S , s.t. (m', w) and

Say w' is match of m in S .

GS is man optimal $\Rightarrow \boxed{w >_m w'}$

(m, w) is unstable for S .



[name entities/objects and study the interaction].