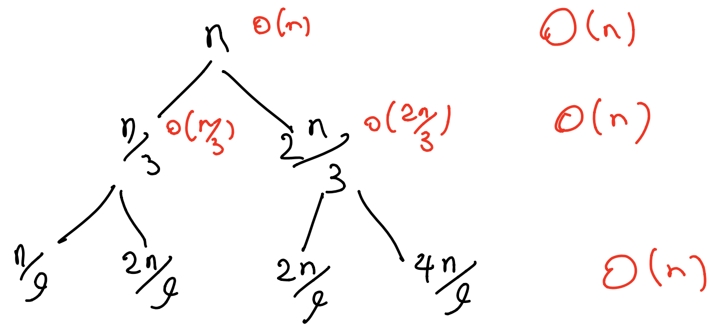
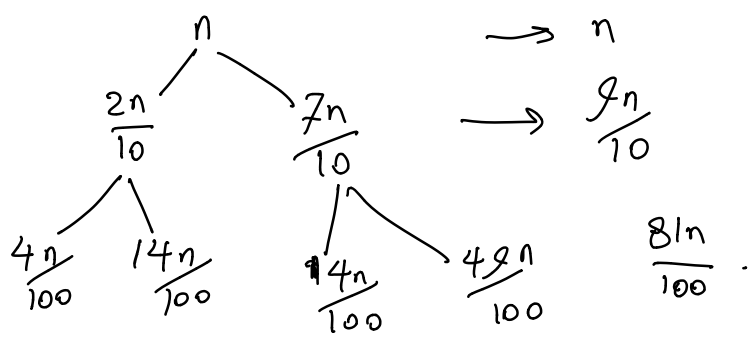
$$T(n) = T(n/3) + T(2n/3) + O(n) \rightarrow O(n \lg n)$$

$$T(n) = T(n/5) + T(7n/10) + O(n) \rightarrow O(n).$$



We will pay n for every row. There are $O(\lg n)$ rows

$\Rightarrow n \lg n$.



The i-th row sums up to $\left(\frac{9}{10}\right)^i \cdot n$.

$$\sum_{i=0}^{\lg n} \left(\frac{9}{10}\right)^i = \frac{(9/10)^{\lg n} - 1}{9/10 - 1} \leq \frac{-1}{-1/10} = 10.$$

Thm: $$\sum_{i=1}^{d} x^i = \frac{x^{d+1} - 1}{x - 1}$$

Thm: Greedy (2) is at most twice OPT.

Supp Greedy (2) chooses $e_1 \ldots e_k$.

Greedy has $2k$ vertices.

We need to show $OPT \geq k$.

Idea: To show OPT chooses at least one endpoint of each e_i. OPT is a vertex cover. So it has to cover $e_1 \dots e_k$ so it chooses at least one endpoint of each e_i.

Claim. Endpoints of e_i's are distinct.

Pf. If e_i, e_j share endpoints (for $j > i$) then we have covered e_j when we add endpoints of i to vertex cover. So Greedy algorithm doesn't process e_j as uncovered edge