**Lem:** Suppose the label of a root is \( k \). There are at least \( 2^k \) nodes in that component/subtree.

**Pf.** At step \( j \) of ALG, any tree has \( \geq 2^{\text{label-root}} \) nodes.

Base: At step 0, all nodes are single comp \( \Rightarrow 1 \) node.

I.H. Suppose at step \( j \). For any tree we have at least \( 2^{\text{label-root}} \) nodes in subtree.

**I.S.** Show it at step \( j+1 \).

**Merge** \( T_a, T_b \).

By I.H \( T_a \) has \( \geq 2^a \) nodes 
\( T_b \) has \( \geq 2^b \) nodes.

Case 1: \( a > b \) bar points to \( a \), no update on labels. New tree has 
\( \geq 2^a \cdot 2^b \geq 2^{a+b} \) nodes.

Case 2: \( a = b \).

Increase \( b \) by 1.

New tree has \( \geq 2^a \cdot 2^{b+1} = 2^{a+b+1} \) nodes.