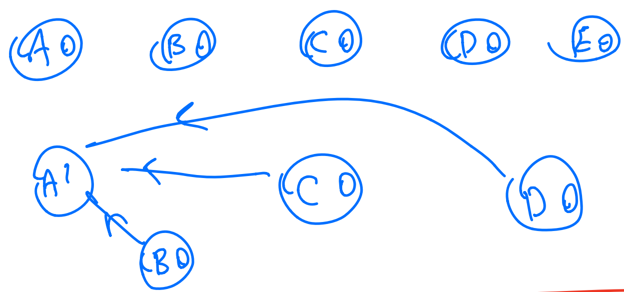



**Lem:** Suppose the label of a root is $k \Rightarrow$ There are at least $2^k$ nodes in that compnent/subtree

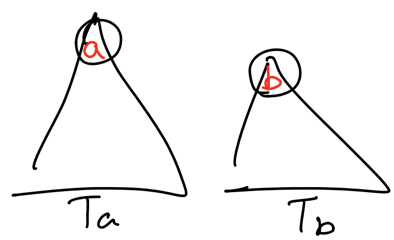
**Pf.** At step $j$ of ALG, Evy tree has $\geq 2^{\text{label-root}}$ nodes.

Base At step 0, all nods are single comp $\Rightarrow$ 1 nods.

I H. Supp at step $j$. For evy tree we have at least $2^{\text{label-root}}$ nodes in subtree.

IS. Show it at step $j+1$.

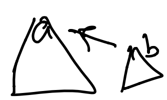
Merge $T_a$, $T_b$.



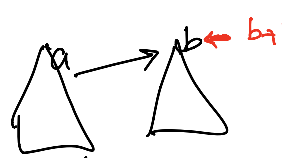

By IH $T_a$ has $\geq 2^a$ nodes

$T_b$ has $\geq 2^b$ nodes

Case 1: $a > b$ b points to a, no update on labels. New tree has $\geq 2^a + 2^b \geq 2^a$ nodes.

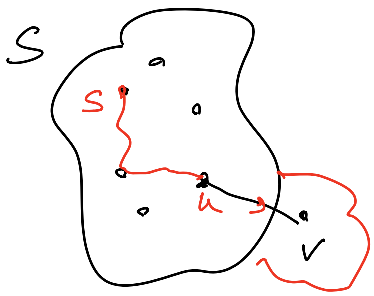



Case 2: $a = b$.




Increase b by 1.

New tree has $\geq 2^a + 2^b = 2 \cdot 2^b = 2^{b+1}$ nodes.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