

Gale Shapley ALG: example

$m_1$   $w_1 > w_2 > w_3$

$m_2$   $w_1 > w_2 > w_3$

$m_3$   $w_2 > w_3 > w_1$

$w_1$   $m_2 > m_1 > m_3$

$w_2$   $m_2 > m_1 > m_3$

$w_3$   $m_2 > m_1 > m_3$

$(m_1, w_1) \times$

$(m_2, w_1)$   $(m_1, w_2)$

$w_2$  rejects  $m_3$

$(m_3, w_3)$

Claim: Output of GS ALG is a perfect matching.

pf. [proof by contradiction]

$p \Rightarrow q$

$\neg q \Rightarrow \neg p$

Assum output is not a P.M.

$\exists$  a man  $m$  who gets no woman.

OBS 1  $\Rightarrow m$  has proposed and got rejected by all women.

$\exists$  a free woman  $w$ .

OBS 3  $\Rightarrow w$  never got engaged  $\Rightarrow m$  has not proposed to  $w$ .

contradiction

$\square$

Claim: Say  $S^*$  is output of GS ALG.  $S^*$  is stable.

pf. [by contradiction]

Assum  $\exists (m, w)$  unstable pair.

-  $m$  prefers  $w$  to his match in  $S^*$

$\rightarrow -w \sim m \sim \text{her} \sim \sim$

OBS 1 m has proposed to w and must have been rejected.

BC m has a worse match at the end.

OBS 3. when w rejected m she had a better option  
and women only trade up. So w prefers her match  
to m  $\Rightarrow (m, w)$  is not unstable