CSE421: Design and Analysis of Algorithms May 30th, 2019

Homework 8

Shayan Oveis Gharan Due: June 6th, 2019 at 5:00 PM

P1) Give a polynomial time algorithm to find the minimum vertex cover in a bipartite graph.

a) Construct a flow network from the input bipartite graph just as in the maximum matching algorithm.

b) Show that every min-cut in this flow network gives a vertex cover whose size is the same as the capacity of the cut.

c) Show that every minimum sized vertex cover in the bipartite graph gives a cut whose capacity is the same as the size of the vertex cover.

d) Write down the algorithm and prove that it works.

P2) Given an $n \times n$ chess board where some cells are removed. Design a polynomial time algorithm to find the maximum number of knights that can be placed on this board such that no two knights attack each other.

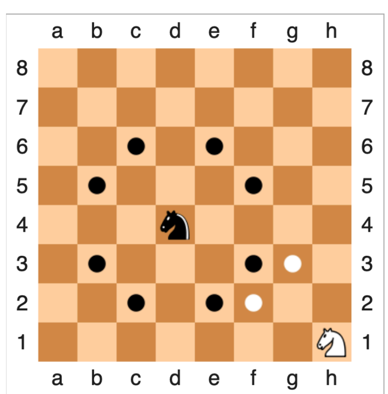
For example in the following 3×3 chessboard the removed cells are marked with X. You can put 4 knights such that no two can attack each other as we did in the right. The location of every knight is marked with a $\bullet$.

X		
		X
	X	

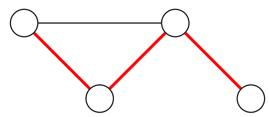
$\Rightarrow$

X		$\bullet$
	$\bullet$	X
$\bullet$	X	$\bullet$

Please see the following image for locations that a knight can attack. In general a knight can attack at most 8 cells if they exist. For example, the white knight can only attack two cells in the following picture.



P3) A Hamiltonian cycle in a graph with n vertices is a cycle of length n, i.e., it is a cycle that visits all vertices exactly once and returns back to the starting point. A Hamiltonian path in a graph with n vertices is a path of length $n-1$, i.e., it is a path that visits all vertices of the graph exactly once. For example, the following graph has a Hamiltonian path marked in red but no Hamiltonian cycle.



Hamiltonian-cycle problem is defined as follows: Given a graph $G = (V, E)$, does it have a Hamiltonian cycle?

Hamiltonian-path problem is defined as follows: Given a graph $G = (V, E)$, does it have a Hamiltonian path?

Prove that Hamil-path $\leq_P$ Hamil-cycle.

P4) **Extra Credit:** Prove that the Hamiltonian cycle problem in directed graphs is NP-Complete. You may use the fact that 3SAT is NP-Complete.