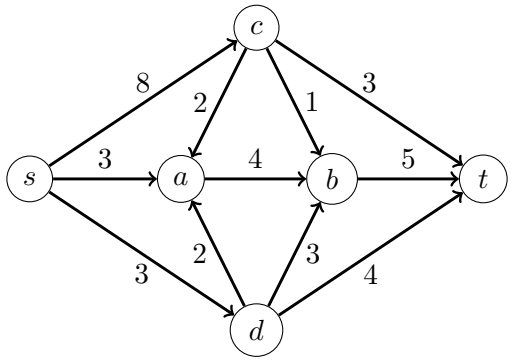
CSE421: Design and Analysis of Algorithms May 23, 2019

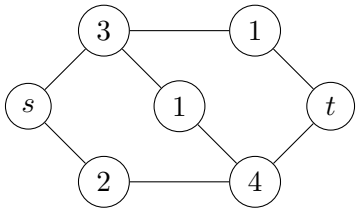
Homework 7

Shayan Oveis Gharan Due: May 30th, 2019 at 5:00 PM

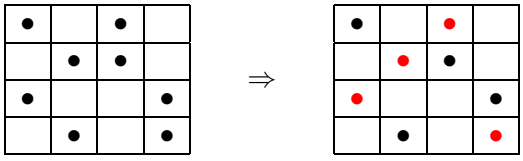
P1) Draw out a maximum s-t flow for the graph below, and the corresponding residual graph G_f. What is the minimum cut that corresponds to this max flow?



P2) Given an undirected graph G and a source vertex s and a destination vertex t. Suppose that each vertex v of G has a capacity $c_v \geq 0$ which shows the maximum amount of flow that can go through v. You can assume that s, t have no capacity. Design a polynomial time algorithm that determines the maximum flow that can be sent from s to t satisfying all capacity constraints of the vertices. For example, in the following picture the maximum amount of flow that can be sent form s to t is 4.



P3) Given a $2k \times 2k$ chess board, there are k rooks in each row and each column of the board. Show that there exist $2k$ rooks such that no two belong to the same row and no two belong to the same column. In the following example (at left), the location of each rook is marked with a $\bullet$, here $k = 2$. In the right you can see $2k$ rooks chosen in red such that no two belong to the same row and no two belong to the same column.



P4) **Extra Credit:** You are given an $m \times n$ array of real numbers. Suppose that the numbers in each row add up to an integer and the numbers in each column add up to an integer. You want to substitute each number $A[i, j]$ with $\lfloor A[i, j] \rfloor$ or $\lceil A[i, j] \rceil$ such that the sum of the numbers in each row and each column remain invariant. Design a polynomial time algorithm that outputs the integer array.

For example, if the input is the left table you can output the right table. Note the sum of numbers in each row (and each column) of the left table is the same as the sum of the numbers of the same row (resp. the same column) in the right table.

0.4	0.1	1.5
0.6	1.9	0.5

$\Rightarrow$

1	0	1
0	2	1