CSE421: Design and Analysis of Algorithms April 18, 2019

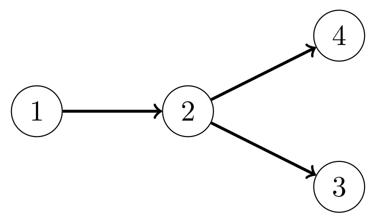
Homework 3

Shayan Oveis Gharan Due: April 25, 2019 at 5:00 PM

P1) We say a directed graph G is strongly connected if for every pair of vertices u, v there is a directed path from u to v and a directed path from v to u. For example, in the following picture the left graph is strongly connected but the right one is not. Design an algorithm that

runs in time $O(m + n)$ and outputs "yes" if G is strongly connected and "no" otherwise.

P2) Given a sequence $d_1, \ldots, d_n$ of integers design a polynomial time algorithm that construct a outward-rooted tree such that the out-degree of vertex i is d_i. An outward-rooted tree is a directed tree where the is a path from root to each vertex. If no such tree exists your algorithm must output "Impossible", otherwise output the edges of the tree. For example, given $1, 2, 0, 0$, we can construct the following tree:



Hint: Show that for every sequence $d_1, \ldots, d_n$ of integers there exists a outward-rooted tree where the out-degree of i is d_i if and only if $\sum_i d_i = n - 1$ and for all i, we have $d_i \geq 0$.

P3) You have n jobs, labeled $1, \ldots, n$, which must be run one at a time, on a single processor. Job j takes time t_j to be processed. We will assume that no two jobs have the same processing time; i.e., $t_i \neq t_j$ for all $i \neq j$. You must decide on a schedule: the order in which to run the jobs. Having fixed an order, each job j has a completion time C_j under this order: this is the total amount of time that elapses (from the beginning of the schedule) before it is done being processed. For example, suppose you have a set of three jobs $\{1, 2, 3\}$ with $t_1 = 3, t_2 = 1, t_3 = 5$, and you run them in this order. Then the completion time of job 1 will be 3, the completion of job 2 will be $3 + 1 = 4$, and the completion time of job 3 will be $3 + 1 + 5 = 9$. Give a polynomial-time algorithm that takes the n processing times $t_1, \ldots, t_n$, and orders the jobs so that the sum of the completion times of all jobs is as small as possible.

P4) You are given a graph G with n vertices and m edges, and a minimum spanning tree T of the graph. Suppose we add a new edge e with weight $w(e)$ to G; call this new graph G'. Give an algorithm that runs in time $O(n)$ to test if T is the minimum spanning tree of G'. Your algorithm should output "yes" if T is still the MST and "no" otherwise. You may assume that all edge weights are distinct.

P5) **Extra Credit:** Suppose G is a 3-colorable graph with n vertices, i.e., it is possible to color the vertices of G with 3 colors such that the endpoints of every edge have distinct colors. Design a polynomial time algorithm that colors vertices of G with $O(\sqrt{n})$ many colors.