Announcements

- Final exam,
  - Monday, December 9, 2:30-4:20 pm
  - Comprehensive (2/3 post midterm, 1/3 pre midterm)
  - Old finals / answers on course website
  - Material covered in lecture
    - Kleinberg, Tardos, Sections 1.1 – 8.10
  - Unlikely to be on the exam
    - 2.5, 4.8, 5.6, 6.9, 7.3, 7.4, 7.13, 8.9

Coping with NP-Completeness

- Approximation Algorithms
- Exact solution via Branch and Bound
- Local Search

 Multiprocessor Scheduling

- Unit execution tasks
- Precedence graph
- K-Processors

  - Polynomial time for \( k=2 \)
  - Open for \( k = \text{constant} \)
  - NP-complete is \( k \) is part of the problem

Highest level first is 2-Optimal

Choose \( k \) items on the highest level

Claim: number of rounds is at least twice the optimal.

Christofides TSP Algorithm

- Undirected graph satisfying triangle inequality
Christofides Algorithm

Bin Packing
- Given N items with weight $w_i$, pack the items into as few unit capacity bins as possible
- Example: .3, .3, .3, .3, .4, .4

First Fit Packing
- First Fit
  - Theorem: FF(I) is at most $17/10 \text{Opt}(I) + 2$
- First Fit Decreasing
  - Theorem: FFD(I) is at most $11/9 \text{Opt}(I) + 4$

Branch and Bound
- Brute force search – tree of all possible solutions
- Branch and bound – compute a lower bound on all possible extensions
  - Prune sub-trees that cannot be better than optimal

Branch and Bound for TSP
- Enumerate all possible paths
- Lower bound: Current path cost plus MST of remaining points
- Euclidean TSP
  - Points on the plane with Euclidean Distance
  - Sample data set: State Capitals

Local Optimization
- Improve an optimization problem by local improvement
  - Neighborhood structure on solutions
  - Travelling Salesman 2-Opt (or K-Opt)
  - Independent Set Local Replacement
What we don’t know

• P vs. NP

If P $\neq$ NP, is there anything in between

• Yes, Ladner [1975]
• Problems not known to be in P or NP Complete
  – Factorization
  – Discrete Log
  – Graph Isomorphism

Complexity Theory

• Computational requirements to recognize languages
• Models of Computation
• Resources
• Hierarchies

Time complexity

• P: (Deterministic) Polynomial Time
• NP: Non-deterministic Polynomial Time
• EXP: Exponential Time

Space Complexity

• Amount of Space (Exclusive of Input)
• L: Logspace, problems that can be solved in $O(\log n)$ space for input of size $n$
  – Related to Parallel Complexity
• PSPACE, problems that can be required in a polynomial amount of space

So what is beyond NP?
### NP vs. Co-NP
- Given a Boolean formula, is it true for some choice of inputs
- Given a Boolean formula, is it true for all choices of inputs

### Problems beyond NP
- Exact TSP, Given a graph with edge lengths and an integer K, does the minimum tour have length K
- Minimum circuit, Given a circuit C, is it true that there is no smaller circuit that computes the same function as C

### Polynomial Hierarchy
- **Level 1**
  - $\exists X_1 \Phi(X_1), \forall X_1 \Phi(X_1)$
- **Level 2**
  - $\forall X_1 \exists X_2 \Phi(X_1, X_2), \exists X_1 \forall X_2 \Phi(X_1, X_2)$
- **Level 3**
  - $\forall X_1 \exists X_2 \forall X_3 \Phi(X_1, X_2, X_3), \exists X_1 \forall X_2 \exists X_3 \Phi(X_1, X_2, X_3)$

### Polynomial Space
- **Quantified Boolean Expressions**
  - $\exists X_1 \forall X_2 \exists X_3 \ldots \exists X_{n-1} \forall X_n \Phi(X_1, X_2, X_3, \ldots, X_{n-1}, X_n)$
- **Space bounded games**
  - Competitive Facility Location Problem
  - N x N Chess
- **Counting problems**
  - The number of Hamiltonian Circuits