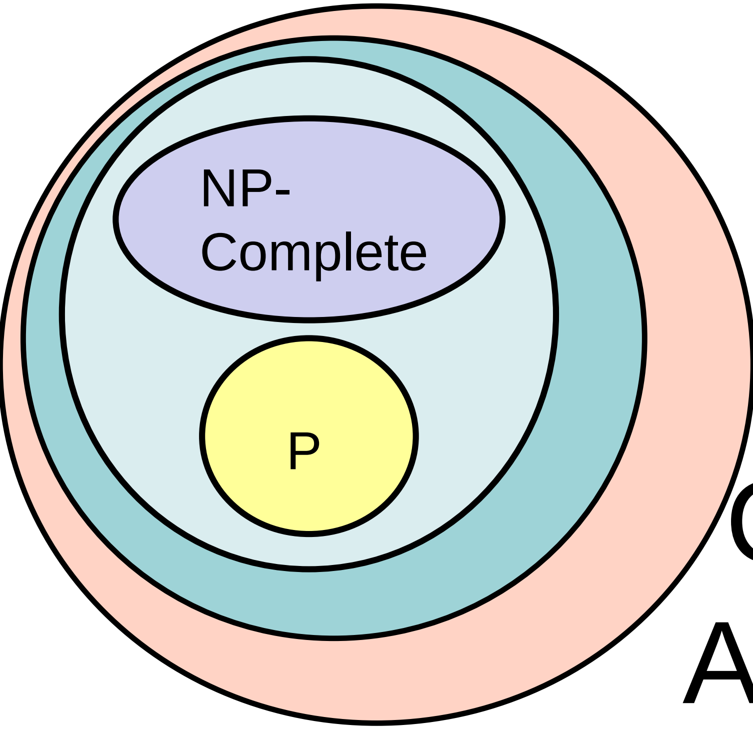


CSE 421 Algorithms

Richard Anderson

Lecture 29

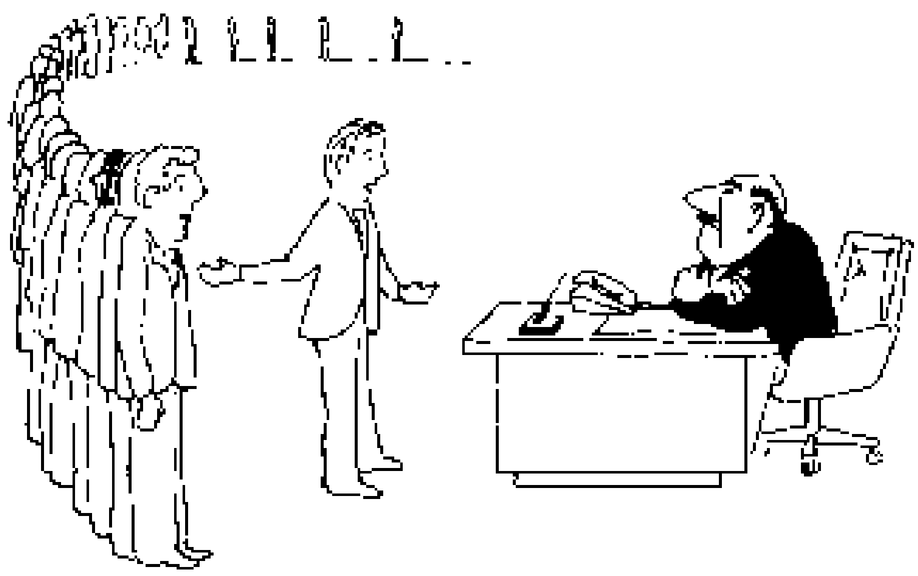
Coping with NP-Completeness
Complexity Theory

Announcements

- Final exam,
  - Monday, December 9, 2:30-4:20 pm
  - Comprehensive (2/3 post midterm, 1/3 pre midterm)
  - Old finals / answers on course website
  - Material covered in lecture
    - Kleinberg, Tardos, Sections 1.1 – 8.10
  - Unlikely to be on the exam
    - 2.5, 4.8, 5.6, 6.9, 7.3, 7.4, 7.13, 8.9

Coping with NP-Completeness

- Approximation Algorithms
- Exact solution via Branch and Bound
- Local Search

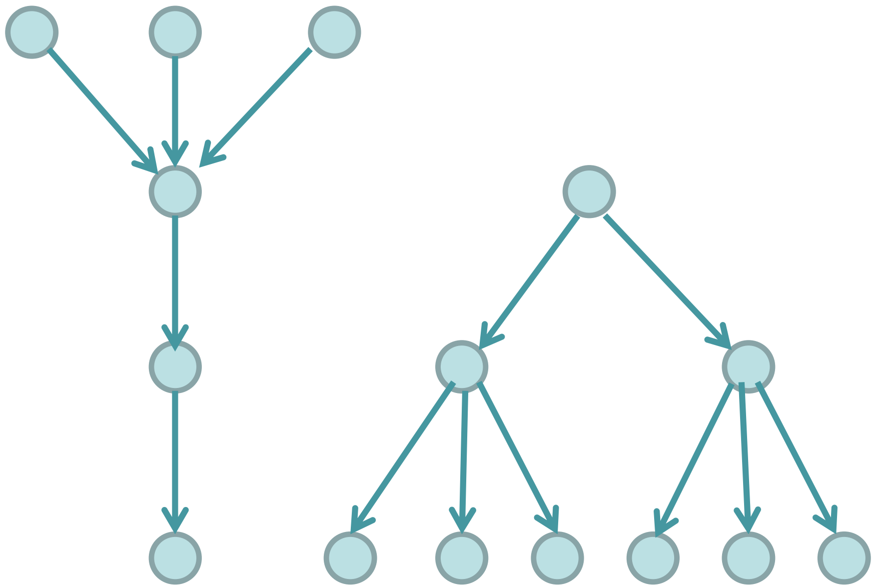


I can't find an efficient algorithm, but neither can all these famous people.

Multiprocessor Scheduling

- Unit execution tasks
- Precedence graph
- K-Processors

- Polynomial time for k=2
- Open for k = constant
- NP-complete is k is part of the problem



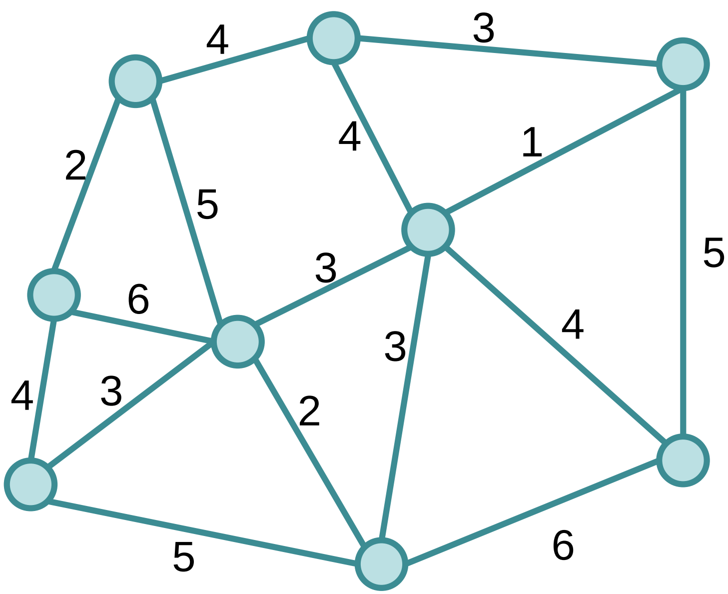
Highest level first is 2-Optimal

Choose k items on the highest level

Claim: number of rounds is at least twice the optimal.

Christofides TSP Algorithm

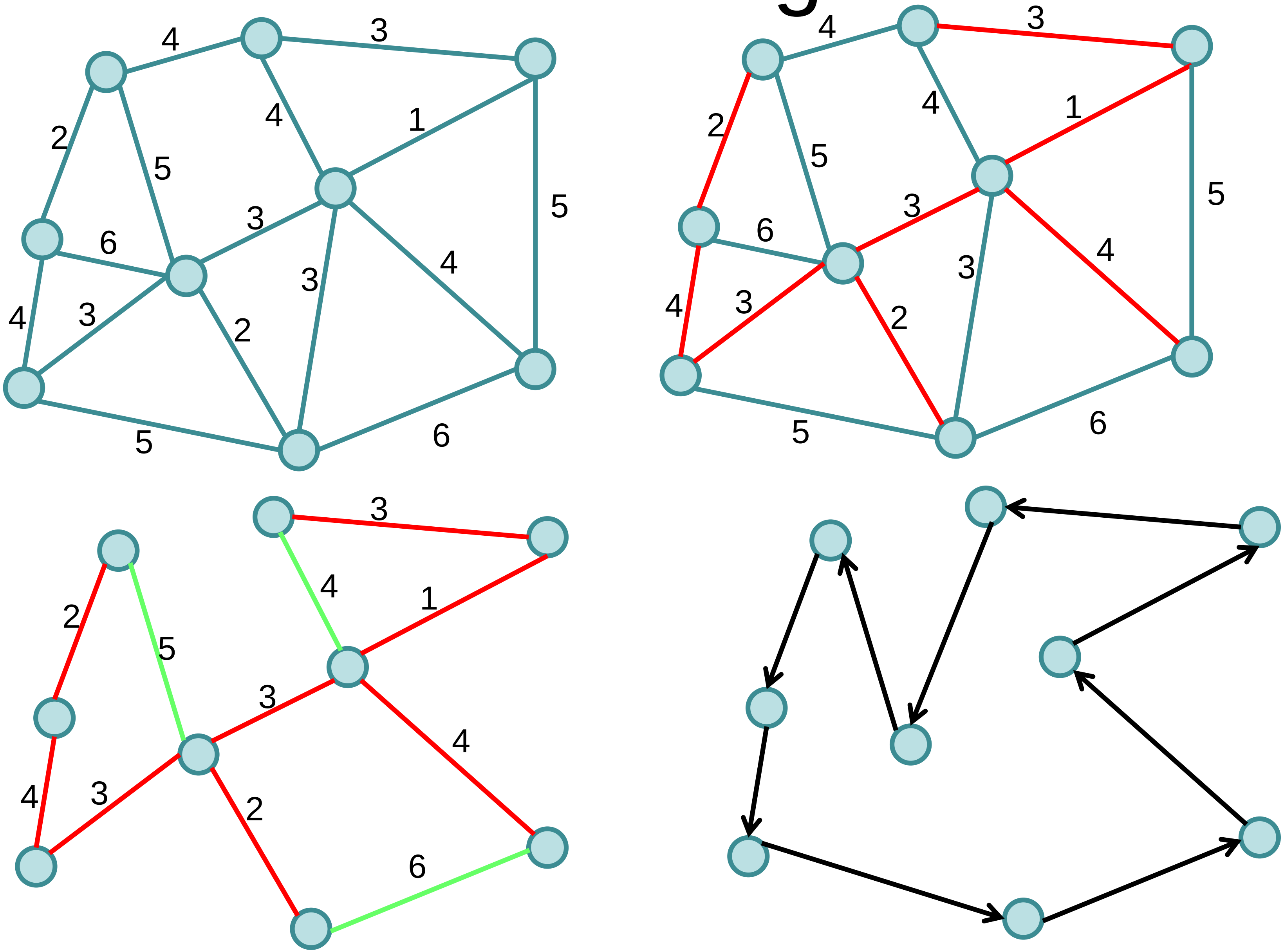
- Undirected graph satisfying triangle inequality



1. Find MST
2. Add additional edges so that all vertices have even degree
3. Build Eulerian Tour

3/2 Approximation

Christofides Algorithm



Bin Packing

- Given N items with weight w_i, pack the items into as few unit capacity bins as possible
- Example: .3, .3, .3, .3, .4, .4

First Fit Packing

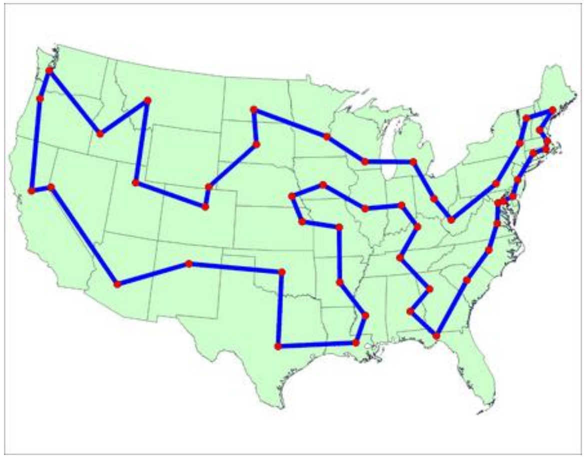
- First Fit
  - Theorem: FF(I) is at most 17/10 Opt(I) + 2
- First Fit Decreasing
  - Theorem: FFD(I) is at most 11/9 Opt (I) + 4

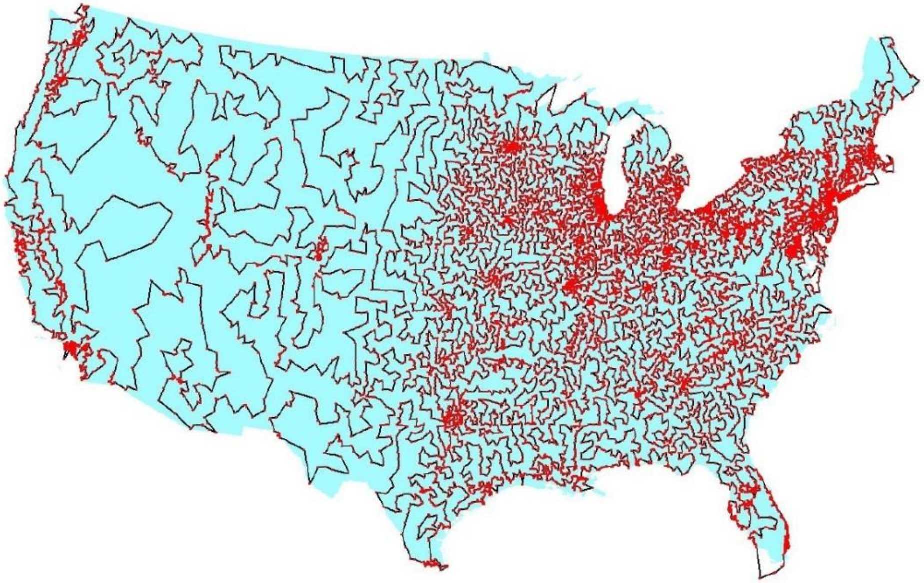
Branch and Bound

- Brute force search – tree of all possible solutions
- Branch and bound – compute a lower bound on all possible extensions
  - Prune sub-trees that cannot be better than optimal

Branch and Bound for TSP

- Enumerate all possible paths
- Lower bound, Current path cost plus MST of remaining points
- Euclidean TSP
  - Points on the plane with Euclidean Distance
  - Sample data set: State Capitals



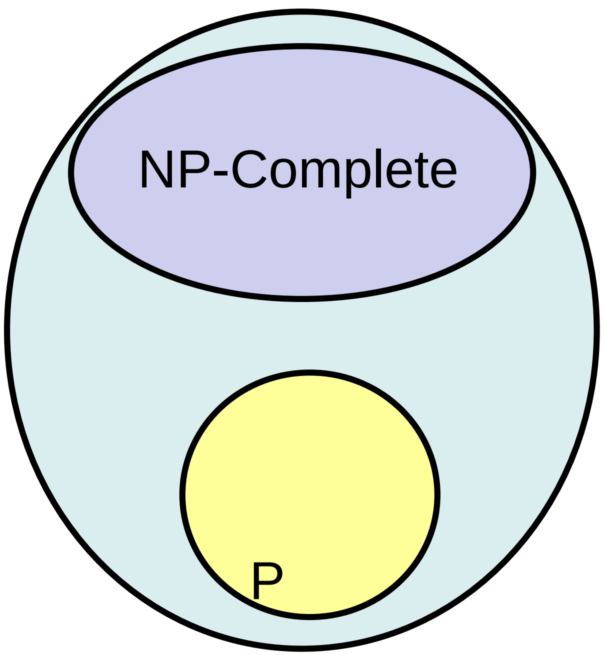


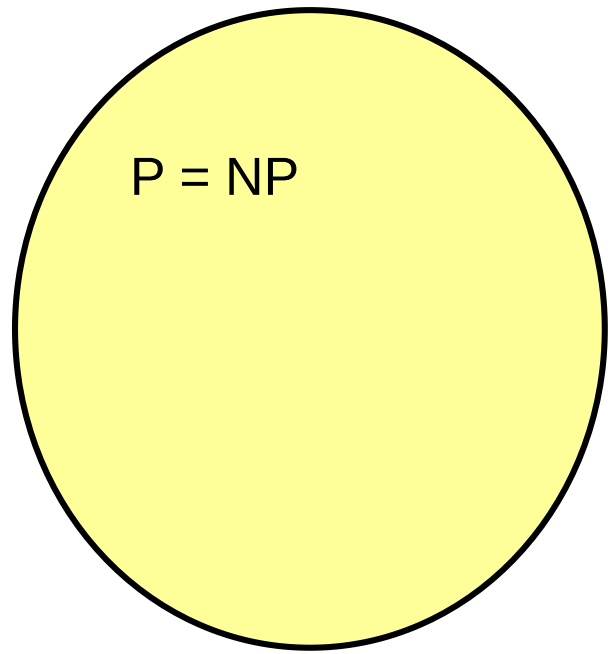
Local Optimization

- Improve an optimization problem by local improvement
  - Neighborhood structure on solutions
  - Travelling Salesman 2-Opt (or K-Opt)
  - Independent Set Local Replacement

What we don't know

- P vs. NP

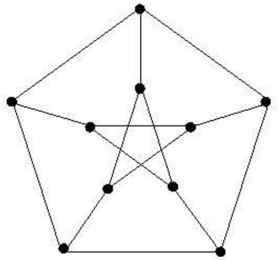


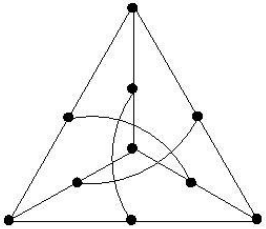


If P != NP, is there anything in between

- Yes, Ladner [1975]
- Problems not known to be in P or NP Complete
  - Factorization
  - Discrete Log
  - Graph Isomorphism

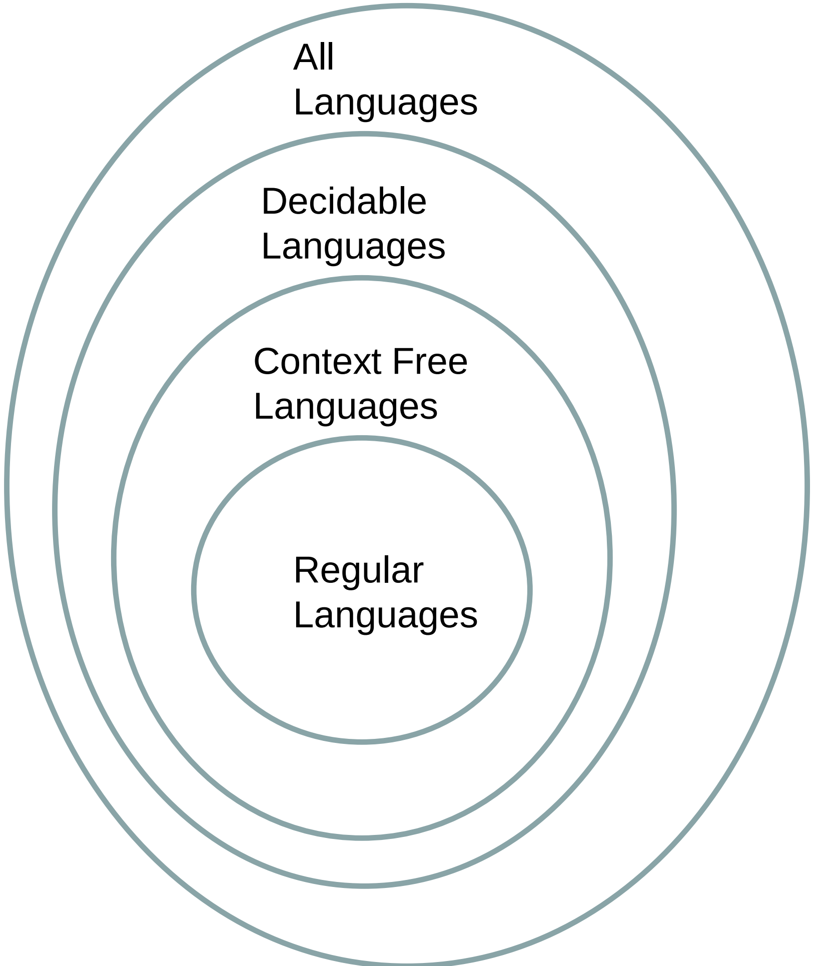
Solve $g^k = b$ over a finite group





Complexity Theory

- Computational requirements to recognize languages
- Models of Computation
- Resources
- Hierarchies



Time complexity

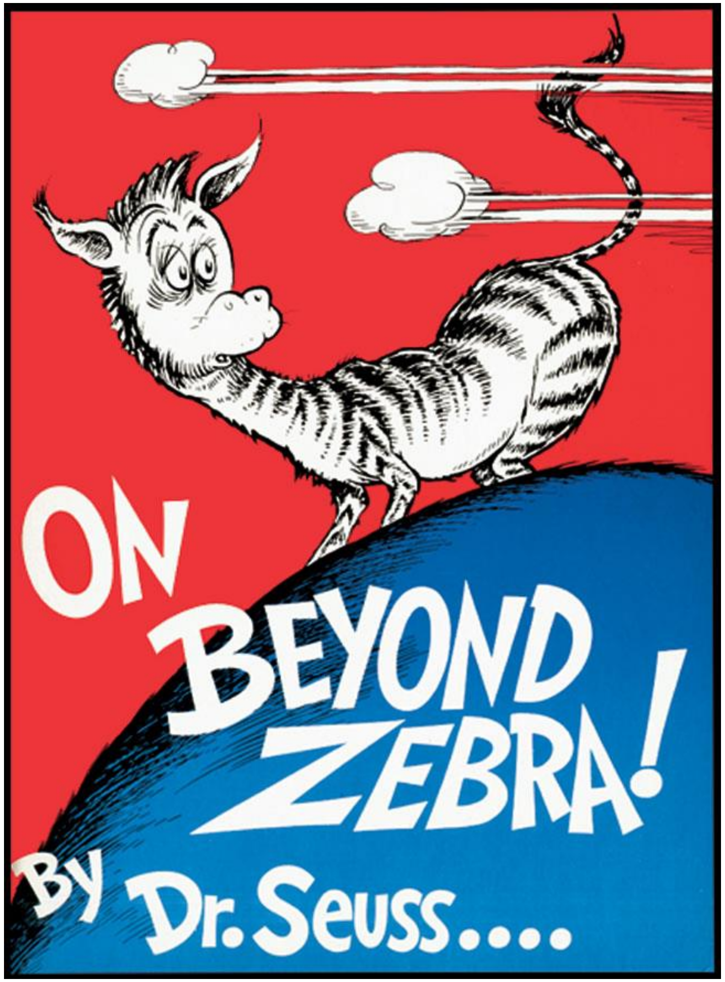
- P: (Deterministic) Polynomial Time
- NP: Non-deterministic Polynomial Time
- EXP: Exponential Time

Space Complexity

- Amount of Space (Exclusive of Input)
- L: Logspace, problems that can be solved in O(log n) space for input of size n
  - Related to Parallel Complexity

- PSPACE, problems that can be required in a polynomial amount of space

So what is beyond NP?



NP vs. Co-NP

- Given a Boolean formula, is it true for some choice of inputs

- Given a Boolean formula, is it true for all choices of inputs

Problems beyond NP

- Exact TSP, Given a graph with edge lengths and an integer K, does the minimum tour have length K

- Minimum circuit, Given a circuit C, is it true that there is no smaller circuit that computes the same function a C

Polynomial Hierarchy

- Level 1
  - $\exists X_1\ \Phi(X_1)$, $\forall X_1\ \Phi(X_1)$
- Level 2
  - $\forall X_1 \exists X_2\ \Phi(X_1,X_2)$, $\exists X_1 \forall X_2\ \Phi(X_1,X_2)$
- Level 3
  - $\forall X_1 \exists X_2 \forall X_3\ \Phi(X_1,X_2,X_3)$, $\exists X_1 \forall X_2 \exists X_3\ \Phi(X_1,X_2,X_3)$

Polynomial Space

- Quantified Boolean Expressions
  - $\exists X_1 \forall X_2 \exists X_3 \ldots \exists X_{n-1} \forall X_n \; \Phi(X_1, X_2, X_3 \ldots X_{n-1} X_n)$
- Space bounded games
  - Competitive Facility Location Problem
  - N x N Chess
- Counting problems
  - The number of Hamiltonian Circuits