Today's topics

• Image Segmentation
• Strip Mining
• Reading: 7.5, 7.6, 7.10-7.12

Minimum Cut Applications

• Image Segmentation
• Open Pit Mining / Task Selection Problem
• Reduction to Min Cut problem

S, T is a cut if S, T is a partition of the vertices with
s in S and t in T
The capacity of an S, T cut is the sum of the capacities of
all edges going from S to T

Image Segmentation

Separate Lion from Savana
Image analysis

- $a_i$: value of assigning pixel $i$ to the foreground
- $b_i$: value of assigning pixel $i$ to the background
- $p_{ij}$: penalty for assigning $i$ to the foreground, $j$ to the background or vice versa
- $A$: foreground, $B$: background
- $Q(A,B) = \sum_{i\in A} a_i + \sum_{j\in B} b_j - \sum_{(i,j)\in E, i\in A, j\in B} p_{ij}$

Pixel graph to flow graph

Mincut Construction

Open Pit Mining

Application of Min-cut

- Open Pit Mining Problem
- Task Selection Problem
- Reduction to Min Cut problem

Open Pit Mining

- Each unit of earth has a profit (possibly negative)
- Getting to the ore below the surface requires removing the dirt above
- Test drilling gives reasonable estimates of costs
- Plan an optimal mining operation
Determine an optimal mine

Generalization

- Precedence graph $G=(V,E)$
- Each $v$ in $V$ has a profit $p(v)$
- A set $F$ is feasible if when $w$ in $F$, and $(v,w)$ in $E$, then $v$ in $F$.
- Find a feasible set to maximize the profit

Min cut algorithm for profit maximization

- Construct a flow graph where the minimum cut identifies a feasible set that maximizes profit

Precedence graph construction

- Precedence graph $G=(V,E)$
- Each edge in $E$ has infinite capacity
- Add vertices $s$, $t$
- Each vertex in $V$ is attached to $s$ and $t$ with finite capacity edges

Find a finite value cut with at least two vertices on each side of the cut
The sink side of a finite cut is a feasible set

- No edges permitted from S to T
- If a vertex is in T, all of its ancestors are in T

Setting the costs

- If $p(v) > 0$:
  - $\text{cap}(v,t) = p(v)$
  - $\text{cap}(s,v) = 0$
- If $p(v) < 0$:
  - $\text{cap}(s,v) = -p(v)$
  - $\text{cap}(v,t) = 0$
- If $p(v) = 0$:
  - $\text{cap}(s,v) = 0$
  - $\text{cap}(v,t) = 0$

Minimum cut gives optimal solution

Why?

Computing the Profit

- $\text{Cost}(W) = \sum_{w \in W; p(w) < 0} -p(w)$
- $\text{Benefit}(W) = \sum_{w \in W; p(w) > 0} p(w)$
- $\text{Profit}(W) = \text{Benefit}(W) - \text{Cost}(W)$

- Maximum cost and benefit
  - $C = \text{Cost}(V)$
  - $B = \text{Benefit}(V)$

Express $\text{Cap}(S,T)$ in terms of $B$, $C$, $\text{Cost}(T)$, $\text{Benefit}(T)$, and $\text{Profit}(T)$

$$\text{Cap}(S,T) = \text{Cost}(T) + \text{Benefit}(S) + \text{Cost}(T) + \text{Benefit}(S) + \text{Benefit}(T) - \text{Benefit}(T)$$

$$= B + \text{Cost}(T) - \text{Benefit}(T) = B - \text{Profit}(T)$$