CSE 421
Algorithms
Autumn 2019
Lecture 24
Network Flow Applications

Announcements
• Homework 9: Due Wednesday, Nov 27
• Homework 10: Due Friday, Dec 6
• Final Exam: Monday, Dec 9, 2:30 PM

Today’s topics
• Network flow reductions
  – Multi source flow
  – Reviewer Assignment
• Baseball Scheduling
• Image Segmentation
• Reading: 7.5, 7.6, 7.10-7.12

Network Flow Definitions
• Flowgraph: Directed graph with distinguished vertices s (source) and t (sink)
• Capacities on the edges, c(e) >= 0
• Problem, assign flows f(e) to the edges such that:
  – 0 <= f(e) <= c(e)
  – Flow is conserved at vertices other than s and t
    • Flow conservation: flow going into a vertex equals the flow going out
  – The flow leaving the source is a large as possible

Key Ideas for Network Flow
• Residual Graph for a Flow
• Augmenting a flow
• Ford Fulkerson Algorithm
• Max Flow / Min Cut Theorem
• Practical Flow Algorithms
• Modelling problems as Network Flow or Minimum Cut

Max Flow / Min Cut
Undirected Network Flow
- Undirected graph with edge capacities
- Flow may go either direction along the edges (subject to the capacity constraints)

Bipartite Matching
- A graph $G=(V,E)$ is bipartite if the vertices can be partitioned into disjoint sets $X,Y$
- A matching $M$ is a subset of the edges that does not share any vertices
- Find a matching as large as possible

Application
- A collection of teachers
- A collection of courses
- And a graph showing which teachers can teach which courses

Converting Matching to Network Flow

Finding edge disjoint paths

Multi-source network flow
- Multi-source network flow
  - Sources $s_1, s_2, \ldots, s_k$
  - Sinks $t_1, t_2, \ldots, t_j$
- Solve with Single source network flow
Resource Allocation: Assignment of reviewers

- A set of papers $P_1, \ldots, P_n$
- A set of reviewers $R_1, \ldots, R_m$
- Paper $P_i$ requires $A_i$ reviewers
- Reviewer $R_j$ can review $B_j$ papers
- For each reviewer $R_j$, there is a list of papers $L_{j1}, \ldots, L_{jk}$ that $R_j$ is qualified to review

Resource Allocation: Illegal Campaign Donations

- Candidates $C_1, \ldots, C_n$
  - Donate $b_i$ to $C_i$
- With a little help from your friends
  - Friends $F_1, \ldots, F_m$
  - $F_i$ can give $a_{ij}$ to candidate $C_j$
  - You can give at most $M_i$ to $F_i$

Baseball elimination

- Can the Dinosaurs win the league?
- Remaining games:
  - AB, AC, AD, AD, AD, BC, BC, BC, BD

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A team wins the league if it has strictly more wins than any other team at the end of the season. A team ties for first place if no team has more wins and there is some other team with the same number of wins.

Assume Fruit Flies win remaining games

- Fruit Flies are tied for first place if no team wins more than 19 games
- Allowable wins
  - Ants (2)
  - Bees (3)
  - Cockroaches (3)
  - Dinosaurs (5)
  - Earthworms (5)
- 18 games to play

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Remaining games


[Diagram of baseball elimination]
Minimum Cut Applications

- Image Segmentation
- Open Pit Mining / Task Selection Problem
- Reduction to Min Cut problem

S, T is a cut if S, T is a partition of the vertices with s in S and t in T
The capacity of an S, T cut is the sum of the capacities of all edges going from S to T

Image Segmentation

- Separate foreground from background

Image analysis

- \( a_i \): value of assigning pixel i to the foreground
- \( b_i \): value of assigning pixel i to the background
- \( p_{ij} \): penalty for assigning i to the foreground, j to the background or vice versa
- A: foreground, B: background
- \( Q(A,B) = \sum_{i \in A} a_i + \sum_{j \in B} b_j - \sum_{(i,j) \in E, i \in A, j \in B} p_{ij} \)

Pixel graph to flow graph

MinCut Construction