CSE 421
Algorithms
Lecture 22
Network Flow, Part 1

Outline
• Network flow definitions
• Flow examples
• Augmenting Paths
• Residual Graph
• Ford Fulkerson Algorithm
• Cuts
• Maxflow-MinCut Theorem

Network Flow Definitions
• Capacity
• Source, Sink
• Capacity Condition
• Conservation Condition
• Value of a flow

Flow Example

Network Flow Definitions
• Flowgraph: Directed graph with distinguished vertices s (source) and t (sink)
• Capacities on the edges, c(e) >= 0
• Problem, assign flows f(e) to the edges such that:
  – 0 <= f(e) <= c(e)
  – Flow is conserved at vertices other than s and t
    • Flow conservation: flow going into a vertex equals the flow going out
  – The flow leaving the source is a large as possible
Flow Example

Find a maximum flow

Residual Graph

- Flow graph showing the remaining capacity
- Flow graph $G$, Residual Graph $G_R$
  - $G$: edge $e$ from $u$ to $v$ with capacity $c$ and flow $f$
  - $G_R$: edge $e'$ from $u$ to $v$ with capacity $c - f$
  - $G_R$: edge $e''$ from $v$ to $u$ with capacity $f$

Augmenting Path Algorithm

- Augmenting path
  - Vertices $v_1, v_2, \ldots, v_k$
    - $v_1 = s$, $v_k = t$
  - Possible to add $b$ units of flow between $v_j$ and $v_{j+1}$ for $j = 1 \ldots k-1$
Augmenting Path Lemma

- Let $P = v_1, v_2, \ldots, v_k$ be a path from $s$ to $t$ with minimum capacity $b$ in the residual graph.
- $b$ units of flow can be added along the path $P$ in the flow graph.

Proof

- Add $b$ units of flow along the path $P$
- What do we need to verify to show we have a valid flow after we do this?

Ford-Fulkerson Algorithm (1956)

while not done
    Construct residual graph $G_r$
    Find an $s$-$t$ path $P$ in $G_r$ with capacity $b > 0$
    Add $b$ units along in $G$

If the sum of the capacities of edges leaving $S$ is at most $C$, then the algorithm takes at most $C$ iterations