CSE 421
Algorithms

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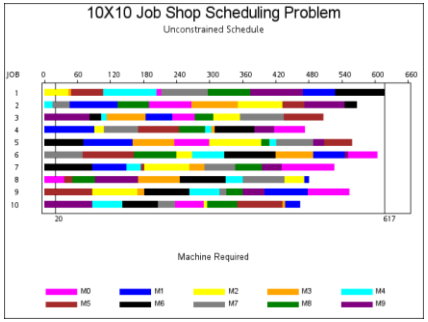
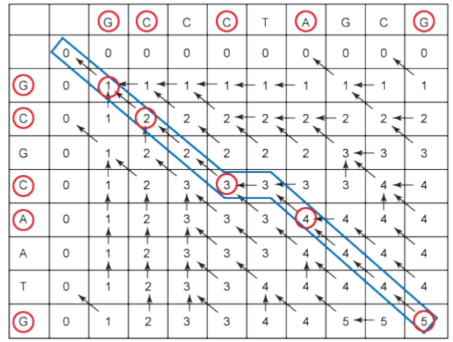
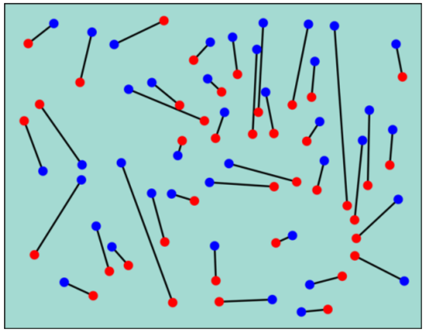
Lecture 4

Announcements

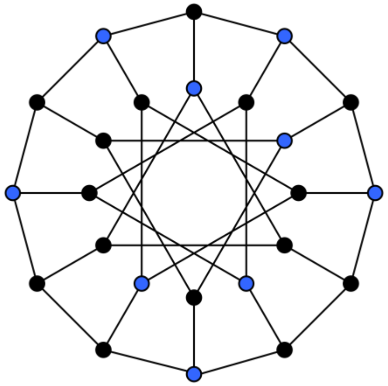
- Guest lecturers Friday and Monday
- Reading
  - Chapter 2.1, 2.2
  - Chapter 3 (Mostly review)
  - Start on Chapter 4
- Homework Guidelines
  - Submit homework with Canvas
    - Submit problems separately
    - Deadline is 1:29 PM on Wednesday
  - Describing an algorithm
    - Clarity is most important
    - Pseudocode generally preferable to just English
      - But sometimes both methods combined work best
  - Prove that your algorithm works
    - A proof is a "convincing argument"
  - Give the run time for your algorithm
    - Justify that the algorithm satisfies the runtime bound
  - You may lose points for style
  - Homework assignments will (probably) be worth the same amount

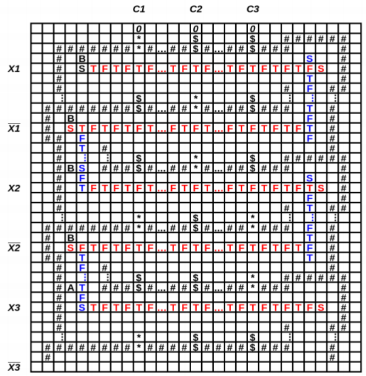






Five Problems





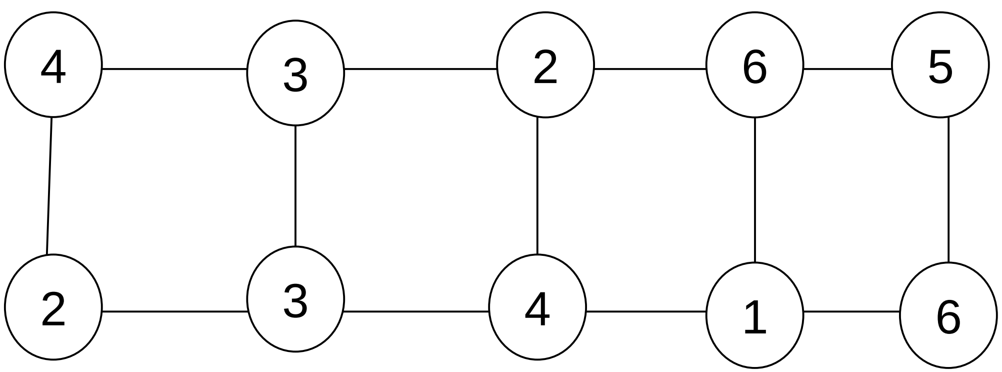
Scheduling
Weighted Scheduling
Bipartite Matching
Maximum Independent Set
Competitive Facility Location

NP-Completenss

- Hard to find a solution
- Easy to verify a solution once you have one
  - Hamiltonian circuit
  - Clique
  - Subset sum
  - Graph coloring

Are there even harder problems?

- Simple game:
  - Players alternating selecting nodes in a graph
    - Score points associated with node
    - Remove nodes neighbors
  - When neither can move, player with most points wins



Competitive Facility Location

- Choose location for a facility
  - Value associated with placement
  - Restriction on placing facilities too close together
  - Competitive placement of facilities
    - E.g., KFC and McDonald’s
- P-Space complete instead of NP-Complete
  - Appear to be much harder
  - No obvious certificate
    - G has a Maximum Independent Set of size 10
    - Player 1 wins by at least 10 points

What does it mean for an algorithm to be efficient?

Definitions of efficiency

- Fast in practice

- Qualitatively better worst case performance than a brute force algorithm

Polynomial time efficiency

- An algorithm is efficient if it has a polynomial run time
- Run time as a function of problem size
  - Run time: count number of instructions executed on an underlying model of computation
  - T(n): maximum run time for all problems of size at most n

Polynomial Time

- Algorithms with polynomial run time have the property that increasing the problem size by a constant factor increases the run time by at most a constant factor (depending on the algorithm)

Why Polynomial Time?

- Generally, polynomial time seems to capture the algorithms which are efficient in practice

- The class of polynomial time algorithms has many good, mathematical properties

Polynomial vs. Exponential Complexity

- Suppose you have an algorithm which takes n! steps on a problem of size n
- If the algorithm takes one second for a problem of size 10, estimate the run time for the following problems sizes:

12 14 16 18 20

Ignoring constant factors

- Express run time as O(f(n))
- Emphasize algorithms with slower growth rates
- Fundamental idea in the study of algorithms
- Basis of Tarjan/Hopcroft Turing Award

Why ignore constant factors?

- Constant factors are arbitrary
  - Depend on the implementation
  - Depend on the details of the model

- Determining the constant factors is tedious and provides little insight

Why emphasize growth rates?

- The algorithm with the lower growth rate will be faster for all but a finite number of cases
- Performance is most important for larger problem size
- As memory prices continue to fall, bigger problem sizes become feasible
- Improving growth rate often requires new techniques

Formalizing growth rates

- $T(n)$ is $O(f(n))$ [$T : Z^+ \rightarrow R^+$]
  - If n is sufficiently large, $T(n)$ is bounded by a constant multiple of $f(n)$
  - Exist c, n_0, such that for $n > n_0$, $T(n) < c\, f(n)$

- $T(n)$ is $O(f(n))$ will be written as:
  $T(n) = O(f(n))$
  - Be careful with this notation

Prove $3n^2 + 5n + 20$ is $O(n^2)$

Let $c =$

Let $n_0 =$

$T(n)$ is $O(f(n))$ if there exist c, n_0, such that for $n > n_0$,
$T(n) < c\, f(n)$

Order the following functions in increasing order by their growth rate

a) $n \log^4 n$
b) $2n^2 + 10n$
c) $2^{n/100}$
d) $1000n + \log^8 n$
e) n^{100}
f) 3^n
g) $1000 \log^{10} n$
h) $n^{1/2}$

Lower bounds

- $T(n)$ is $\Omega(f(n))$
  - $T(n)$ is at least a constant multiple of $f(n)$
  - There exists an n_0, and $\varepsilon > 0$ such that $T(n) > \varepsilon f(n)$ for all $n > n_0$
- Warning: definitions of Ω vary

- $T(n)$ is $\Theta(f(n))$ if $T(n)$ is $O(f(n))$ and $T(n)$ is $\Omega(f(n))$

Useful Theorems

- If lim $(f(n) / g(n)) = c$ for $c > 0$ then $f(n) = \Theta(g(n))$

- If $f(n)$ is $O(g(n))$ and $g(n)$ is $O(h(n))$ then $f(n)$ is $O(h(n))$

- If $f(n)$ is $O(h(n))$ and $g(n)$ is $O(h(n))$ then $f(n) + g(n)$ is $O(h(n))$

Ordering growth rates

- For $b > 1$ and $x > 0$
  - $\log^b n$ is $O(n^x)$

- For $r > 1$ and $d > 0$
  - n^d is $O(r^n)$