University of Washington November 19, 2019
Department of Computer Science and Engineering
CSE 421, Autumn 2019

Homework 9, Due Wednesday, November 27, 2019

Problem 1 (10 points):

Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

Let G be an arbitrary flow network, with a source s, a sink t and a positive integer capacity c_e on every edge e; and let (A, B) be a minimum $s - t$ cut with respect to these capacities $\{c_e : e \in E\}$. Now suppose we add 1 to every capacity; then (A, B) is still a minimum $s - t$ cut with respect to these new capacities $\{1 + c_e : e \in E\}$.

Problem 2 (10 points):

You are given a flow network with unit-capacity edges: it consists of a directed graph $G = (V, E)$, a source $s \in V$, and a sink $t \in V$; and $c_e = 1$ for every $e \in E$. You are also given a parameter k.

The goal is to delete k edges so as to reduce the maximum $s - t$ flow in G by as much as possible. In other words, you should find a set of edges $F \subseteq E$ so that $|F| = k$ and the maximum $s - t$ flow in $G' = (V, E - F)$ is as small as possible subject to this.

Give a polynomial time algorithm to solve this problem, and justify that your algorithm is correct.

Problem 3 (10 points):

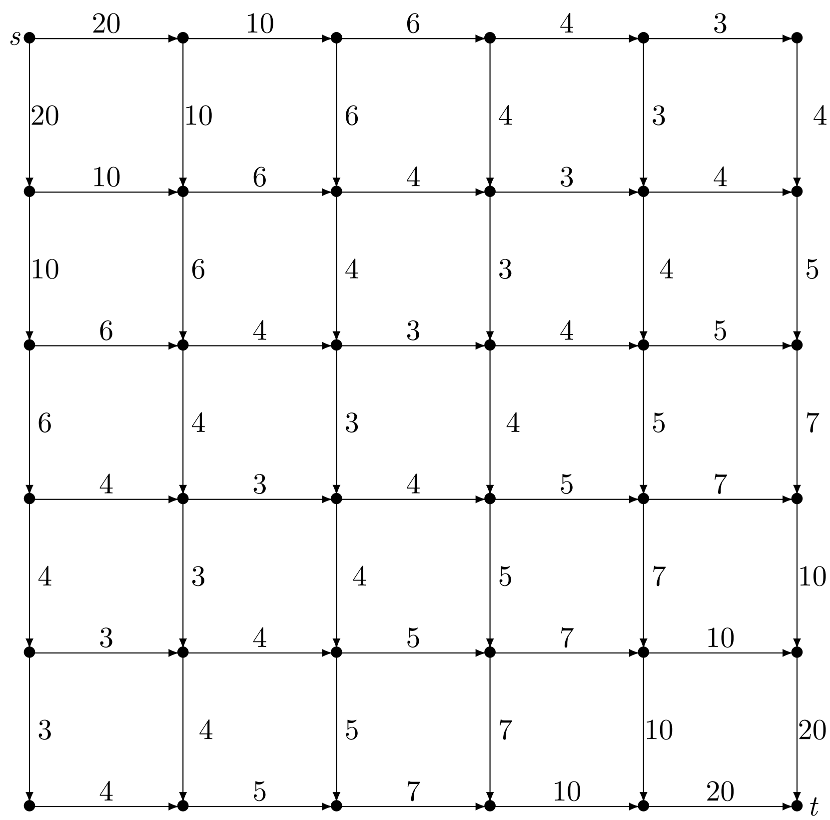
In a standard $s - t$ Maximum-Flow Problem, we assume edges have capacities, and there is no limit on how much flow is allowed to pass through a node. In this problem, we consider the variant of the Maximum-Flow problem with node capacities.

Let $G = (V, E)$ be a directed graph, with source $s \in V$, sink $t \in V$, and nonnegative node capacities $\{c_v \geq 0\}$ for each $v \in V$. Given a flow f in this graph, the flow through a node v is defined as $f^{\text{in}}(v)$, the sum of the flows on the incoming edges to v. We say that a flow is feasible if it satisfies the usual flow-conservation constraints and the node-capacity constraints: $f^{\text{in}}(v) \leq c_v$ for all nodes.

Give a polynomial-time algorithm to find an $s - t$ maximum flow in such a node-capacitated network. Justify the correctness of your algorithm.

Problem 4 (10 points):

Consider the following flow graph. Find a maximum flow.



a) What is the value of the maximum flow? Indicate the value of flow on each edge.

b) Prove that your flow is maximum.

Problem 5 (10 Points):

(Kleinberg-Tardos, Exercise 9, Page 419) Network flow issues come up in dealing with natural disasters and other crises, since major unexpected events often require the movement and evacuation of large numbers of people in a short amount of time.

Consider the following scenario. Due to large-scale flooding in a region, paramedics have identified a set of n injured people distributed across the region who need to be rushed to hospitals. There are k hospitals in the region, and each of the n people needs to be brought to a hospital that is within a half-hour's driving time of their current location (so different people will have different options for hospitals, depending on where they are right now).

At the same time, one does not want to overload any one of the hospitals by sending too many patients its way. The paramedics are in touch by cell phone, and they want to collectively work out whether they can choose a hospital for each of the injured people in such a way that the load on the hospitals is balanced: Each hospital receives at most $\lceil n/k \rceil$ people.

Give a polynomial-time algorithm that takes the given information about the people's locations and determines whether this is possible.