University of Washington
Department of Computer Science and Engineering
CSE 421, Autumn 2019

November 4, 2019

Homework 6, Due Wednesday, November 6, 2019

On all problems provide justification of your answers. For the algorithms problems, provide an clear explanation of why your algorithm solves the problem, as well as a justification of the run time. Since this assignment is from the Divide and Conquer section, express your algorithms in a recursive manner.

Problem 1 (10 points):
Solve the following recurrences:

a)

$$T(n) = \begin{cases} T(\frac{n}{2}) * T(\frac{n}{2}) \\ 2 \end{cases} \quad \text{if } n \leq 1$$

b)

$$T(n) = \begin{cases} T(n-1) * T(n-1) \\ 2 \end{cases} \quad \text{if } n \leq 1$$

Problem 2 (10 points):
Suppose you are working in the quality control of a factory that produces quarters for the US government and your job is to make sure that all quarters have exactly the same weight. You are given 2^k quarters for $k \geq 2$ and you know that at most one of them can be defective. A defective quarter will weight higher or lower than normal. You are given a scale with two trays: Each time you can put a set S of quarters in the left and a set T in the right (for disjoints sets S, T). The scale will show if S is heavier than T, or T is heavier than S, or they have exactly the same weight. Design an algorithm to find the defective quarter (if it exists) by using the scale only $O(k)$ many times. (Note that your algorithm will run by a human not a computer.) Justify your algorithm is correct.

Problem 3 (10 points):
Suppose A is an array of n integers that is a strictly decreasing sequence, followed by a strictly increase sequence such as $[12, 9, 8, 6, 3, 4, 7, 9, 11]$. Give an $O(\log n)$ algorithm to find the minimum element of the array. Justify your algorithm is correct.

Problem 4 (10 points):
Let A and B be two sorted arrays of integers, each of length n. Show how you can find the median of the combined set of elements in $O(\log n)$ comparisons. (As in the Median algorithm discussed in lecture, you will need to solve the Select the k-th largest problem.) Justify your algorithm is correct.