University of Washington
Department of Computer Science and Engineering
CSE 421, Autumn 2019

October 16, 2019

Homework 4, Due Wednesday, October 23, 1:29 pm, 2019

Turnin instructions: Electronics submission on canvas using the CSE 421 canvas site. Each numbered problem is to be turned in as a separate PDF.

Problem 1 (10 points):

Let S be a set of intervals, where $S = \{I_1, \ldots, I_n\}$ with $I_j = (s_j, f_j)$ and $s_j < f_j$. A set of points $P = \{p_1, \ldots, p_k\}$ is said to be a *cover* for S if every interval of S includes at least one point of P, or more formally: for every I_i in S, there is a p_j in P with $s_i \leq p_j \leq f_i$.

Describe an algorithm that finds a cover for S that is as small as possible, and prove that your algorithm finds a minimum size cover. You algorithm should be efficient. In this case $O(n \log n)$ is achievable.

Problem 2 (10 points):

Suppose you are given a connected graph G, with edge costs that are all distinct. Prove that G has a unique minimum spanning tree.

Problem 3 (10 points):

Let $G = (V, E)$ be a directed acyclic graph with lengths assigned to the edges. Give an $O(n + m)$ time algorithm that given vertices $s, t \in V$ finds a maximum length path from s to t. Justify that your algorithm is correct.

Problem 4 (10 points):

Let $G = (V, E)$ be a directed graph with lengths assigned to the edges. Let $\delta(u, v)$ denote the shortest path distance from u to v. Prove that for all vertices $u, v, w \in V$:

$$\delta(u, w) \leq \delta(u, v) + \delta(v, w).$$

You may assume that the graph is strongly connected, so that there is a path between every pair of vertices.

Problem 5 (10 points):

Let $G = (V, E)$ be a connected, undirected graph with weights on the edges. In this problem, the edge costs need not be distinct, so there may be multiple minimum spanning trees. Suppose that T is a spanning tree with the property that every edge $e \in T$ is in *some* minimum spanning tree for G. Is T necessarily a minimum spanning tree? Give a proof or a counterexample with an explanation.

Problem 6 (10 points):

Let $G = (V, E)$ be a directed graph with integral edge costs in $\{1, 2, 3, 4, 5, 6, 7, 8\}$. Give an $O(n+m)$ time algorithm that given vertices $s, t \in V$ finds a shortest path from s to t.