University of Washington
Department of Computer Science and Engineering
CSE 421, Autumn 2019

October 2, 2019

# Homework 2, Due Wednesday, October 9, 1:29 pm, 2019

Turnin instructions: Electronics submission on canvas using the CSE 421 canvas site. Each numbered problem is to be turned in as a separate PDF.

**Problem 1 (10 points):**

Order the following functions in increasing order by their growth rate:

1. $n^3$
2. $(\log n)^{\log n}$
3. $n^{\sqrt{\log n}}$
4. $2^{n/10}$

Explain how you determined the ordering.

**Problem 2 (10 points):**

Prove that $4n^2 + 3n \log n + 6n + 20 \log^2 n + 11$ is $O(n^2)$.

**Problem 3 (10 points):**

Suppose that $f(n)$ is $O(r(n))$ and $g(n)$ is $O(s(n))$. Let $h(n) = f(n)g(n)$ and $t(n) = r(n)s(n)$. Prove that $h(n)$ is $O(t(n))$.

**Problem 4 (10 points):**

Give an algorithm for efficiently computing the *number* of shortest paths in an undirected graph between a a pair of vertices. Suppose that you have an undirected graph $G = (V, E)$ and a pair of vertices $v$ and $w$. Your algorithm should compute the number of shortest $v - w$ paths in $G$. Since this graph is unweighted, the length of a path is defined to be the number of edges in the path. Your algorithm should have run time $O(n + m)$ for a graph of $n$ vertices and $m$ edges.

You should explain why your algorithm is correct and justify the run time of the algorithm.

**Problem 5 (10 points):**

The *diameter* of an undirected graph is the maximum distance between any pair of vertices. If a graph is not connected, its diameter is infinite. Let $G$ be an $n$ node undirected graph, where $n$ is even. Suppose that every vertex has degree at least $n/2$. Prove that $G$ has diameter at most 2.

**Problem 6 (10 points):**

Given an undirected graph $G = (V, E)$ with $n$ vertices such that the degree of every vertex of $G$ is at most $k$. Show that we can color the edges of $G$ with at most $2k - 1$ colors such that any pair of edges $e$ and $f$ which are incident to the same vertex have distinct colors.