University of Washington
Department of Computer Science and Engineering
CSE 421, Autumn 2019

November 26, 2019

Homework 10, Due Friday December 6, 2019

Problem 1 (10 points):

Answer the following questions with "yes", "no", or "unknown, as this would resolve the P vs. NP question." Give a brief explanation of your answer.

Define the decision version of the Interval Scheduling Problem as follows: Given a collection of intervals on a time-line, and a bound k, does the collection contain a subset of nonoverlapping intervals of size at least k?

a) Question: Is it the case that Interval Scheduling $\leq_P$ Vertex Cover?

b) Question: Is it the case that Independent Set $\leq_P$ Interval Scheduling?

Problem 2 (10 points):

Suppose that you have an $O(n^3)$ time algorithm for the Hamiltonian Circuit Problem. Prove that $P = NP$.

Problem 3 (10 points):

(Kleinberg-Tardos, Page 505, Problem 3). Suppose you are helping to organize a summer sports camp, and the following problem comes up. The camp is supposed to have at least one counselor who is skilled at each of the n sports covered by the camp (baseball, volleyball, and so on). They have received job applications from m potential counselors. For each of the n sports, there is some subset of the m applications qualified in that sport. The question is: For a given number $k < m$, is it possible to hire at most k of the counselors and have at least one counselor qualified in each of the n sports. We will call this the *Efficient Recruiting Problem.* Show that the Efficient Recruiting Problem is NP-Complete.

Problem 4 (10 points):

(Kleinberg-Tardos, Page 507, Problem 7). Since the 3-Dimensional Matching Problem is NP-complete, it is natural to expect that the corresponding 4-Dimensional Matching Problem is at least as hard. Let us define *4-Dimensional Matching* as follows. Given sets W, X, Y, and Z, each of size n, and a collection C of ordered 4-tuples of the form (w_i, x_j, y_k, z_l), do there exist n 4-tuples from C so that no two have an element in common?
Prove that 4-Dimensional Matching is NP-Complete.

Problem 5 (10 Points):

(Kleinberg-Tardos, Page 513, Problem 17). You are given a directed graph $G = (V, E)$ with weights w_e on its edges $e \in E$. The weights can be negative or positive. The *Zero-Weight-Cycle Problem* is to decide if there is a simple cycle in G so that the sum of the edge weights on this cycle is exactly 0. Prove that the Zero-Weight-Cycle problem is NP-Complete. (Hint: Hamiltonian PATH)