University of Washington October 31, 2016
Department of Computer Science and Engineering
CSE 421, Autumn 2016

Midterm Exam, Monday, October 31, 2016

NAME: ______________________________

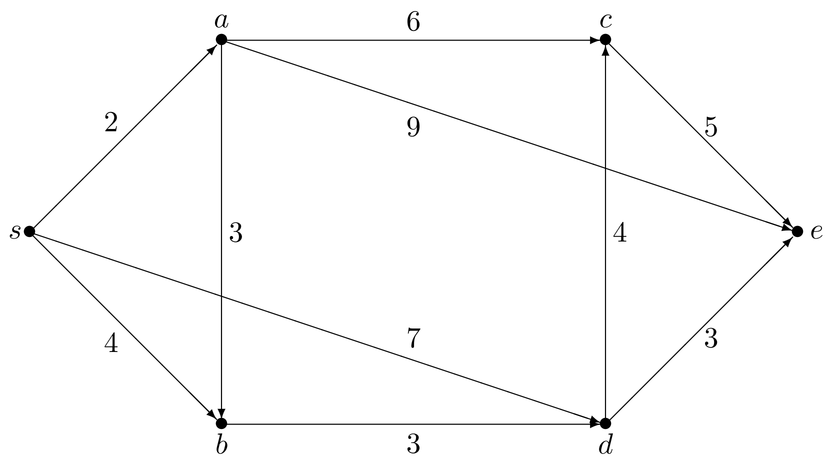
Instructions:

- Closed book, closed notes, no calculators
- Time limit: 50 minutes
- Answer the problems on the exam paper.
- If you need extra space use the back of a page
- Problems are not of equal difficulty, if you get stuck on a problem, move on.

1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
Total	/60

Problem 1 Dijkstra's (10 points):

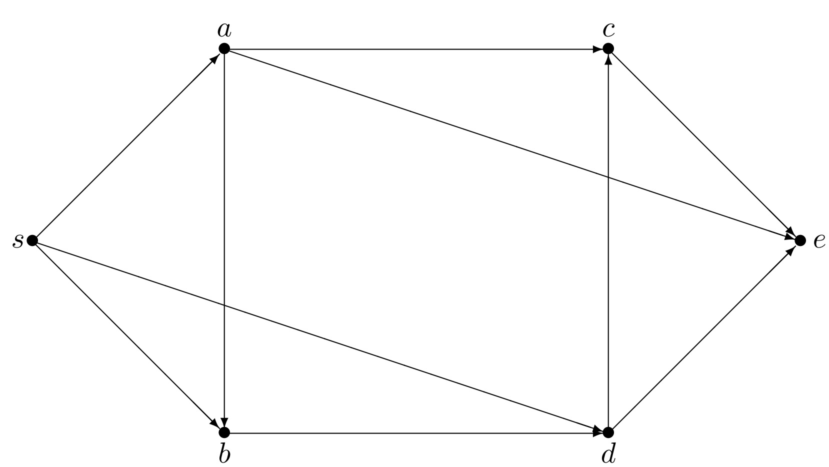
Use the following graph to simulate versions of Dijkstra's algorithm in parts a) and c) starting from the vertex s.

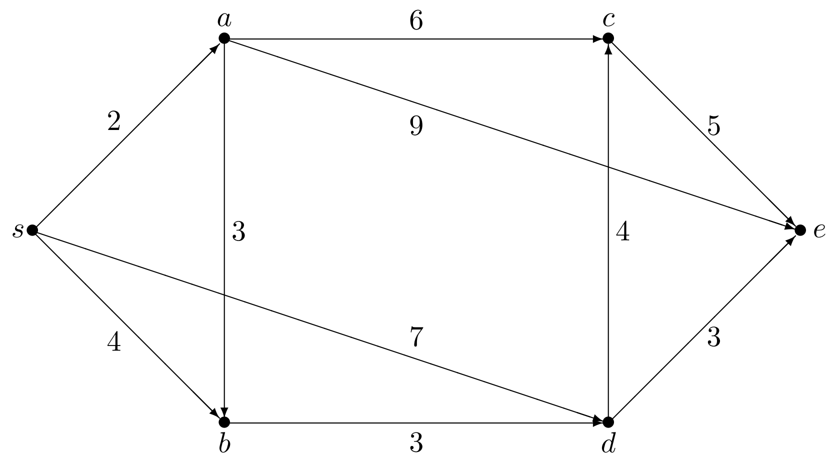


a) Simulate Dijkstra's shortest path algorithm on the graph above by filling in the table. The entries should contain the preliminary distance values.

Round	Vertex	s	a	b	c	d	e
1							
2							
3							
4							
5							
6							

b) Draw the back edges found by your simulation of Dijkstra's algorithm.

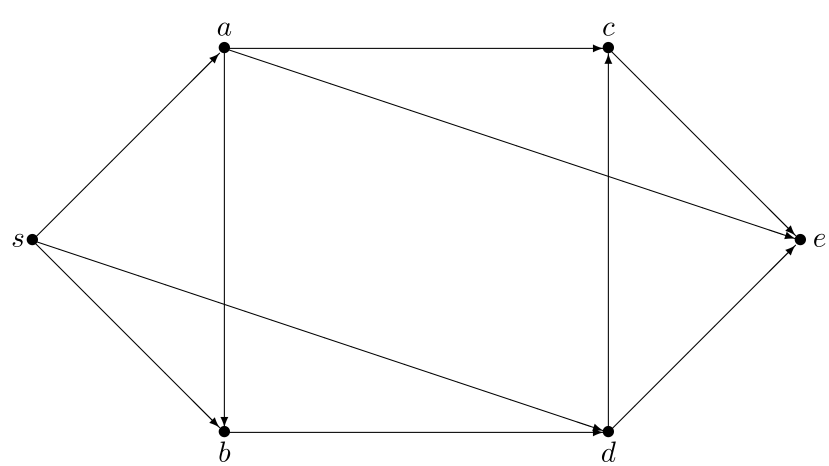




c) Simulate Dijkstra's bottleneck path algorithm on the graph above by filling in the table. The entries should contain the preliminary distance values.

Round	Vertex	s	a	b	c	d	e
1							
2							
3							
4							
5							
6							

d) Draw the back edges found by your simulation of Dijkstra's bottleneck path algorithm.



Problem 2 Fun with Big-Oh (10 points):

a) Order the following functions in increasing order by their growth rate:

1. $(\log n)^{\log n}$
2. n^4
3. $n^3 + n^5$
4. $2^{\sqrt{\log n}}$
5. $(0.01)^n$
6. $2^{n/10}$

b) Is $n^2 \in \Theta(3n^3 + 2n)$? Explain.

c) Is an algorithm with run time $O(n!)$ ever preferable to an algorithm with runtime $O(n)$? Explain.

Problem 3 Minimum Spanning Tree (10 points):

Let $G = (V, E)$ be a connected, undirected graph with edge weights. The edge weights are not necessarily distinct (e.g., the graph may have two or more edges of the same weight).

a) Let e be a minimum weight edge in E. Prove that e is not necessarily in every minimum spanning tree for G.

b) Let e be a minimum weight edge in E. Prove that e is in some minimum spanning tree for G.

Problem 4 Starbucks Placement (10 points):

Washington State law requires that there is a Starbucks within 15 miles when driving on the freeway.[1] The purpose of this problem is to design an algorithm that Starbucks can use to place it's stores along a freeway to ensure that all points are within a fixed distance of a store. The stores can only be placed at off ramps, so there is a list of possible locations given as an increasing sequence of integers. The different directions of the freeway are considered separately, and no back tracking is allowed to reach a store. The problem is: Given a set of integers $A = \{a_1, a_2, \ldots a_n\}$ in increasing order, find a subset S of A which is as small as possible, such that for every $x \in [0, K]$, there is an $a \in S$ with $a - L \leq x \leq a$ where K is the length of the freeway, and L is the maximum allowed distance from a Starbucks.

a) Give a greedy algorithm that finds a minimum sized set of locations that guarantees every point is with distance L of a Starbucks.

b) Prove that the first item that your greedy algorithm selects is a member of some optimal solution to the problem.

[1] This is made up.

Problem 5 Scheduling (10 points):

Let G be the precedence graph (prerequisite graph) for the courses in the Computer Science major. Describe an algorithm for determining the minimum number of quarters to complete the major. (Design the algorithm for over achievers with no bound on the number of courses that can be taken per quarter[2]. Also, assume that every course is offered every quarter.) Justify that your algorithm is correct. You do not need to give the runtime for your algorithm (but it should be a polynomial time algorithm.)

[2] This problem is hard if a bound k is put on the number of courses. No efficient algorithm is known for $k = 3$.

Problem 6 Recurrences (10 points):

Give solutions to the following recurrences. Justify your answers.

a)

$$T(n) = \begin{cases} 4T(\frac{n}{3}) + n & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$

b)

$$T(n) = \begin{cases} 25T(\frac{n}{5}) + n^2 & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$