CSE 421 - Lecture 9

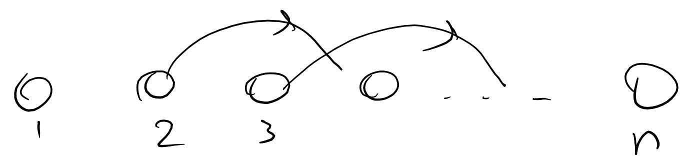
Wednesday, January 24, 2018 2:25 PM

G has a topo order $\rightarrow$ G is a DAG.

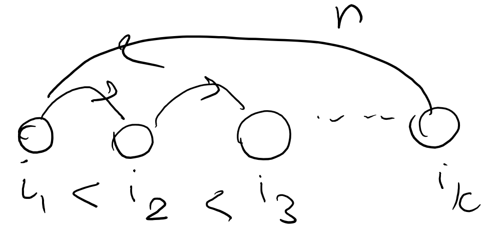
Pf: by contradiction

Suppose G is not a DAG.

It has a cycle C_1.



Another proof.



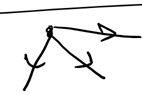
Choose lowest index vertex in cycle

$\underline{\underline{i}}$. $\exists j$ s.t. (j, i) an edge of C_1.

j must come before i because $j \rightarrow i$

i But i is lowest index so $i < j$ a contradiction!

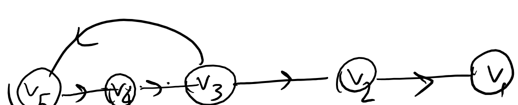
Lemma: If G is a DAG it has a source node.



By contradiction.

Suppose G has no source nodes.

$\Rightarrow$ Every vertex has at least one incoming edge.



[Similar to proof that if $\deg(v) \geq 2$ in G then G has a cycle].

Thm: If G is a DAG $\Rightarrow$ It has a topological order.

By induction

Base Case: A DAG with 1 node $\checkmark$

IH: Every DAG with n-1 nodes has a topol order

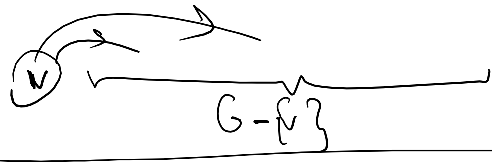
IS: We want to show every DAG with n nodes has topo order

Given a DAG. G.

G has a source node. $\underline{v}$.

$G - \{v\}$ is a DAG BC G was a DAG and we only removed a vertex and edges

$G - \{v\}$ has a topo order.



Thm: Greedy is Optimal.

Pf (Greedy stays ahead):

Say $i_1\ i_2 \cdots i_k$ is output of Greedy

$j_1\ j_2 \cdots j_m$ " " OPT.

Claim: $\forall r \quad f(i_r) \le f(j_r)$.

Induction:

Base $f(i_1) \le f(j_1)$ $\quad i_1$ smallest fin time

IH: $f(i_r) \le f(j_r)$ for s— r.

IS: Goal: $f(i_{r+1}) \le f(j_{r+1})$.