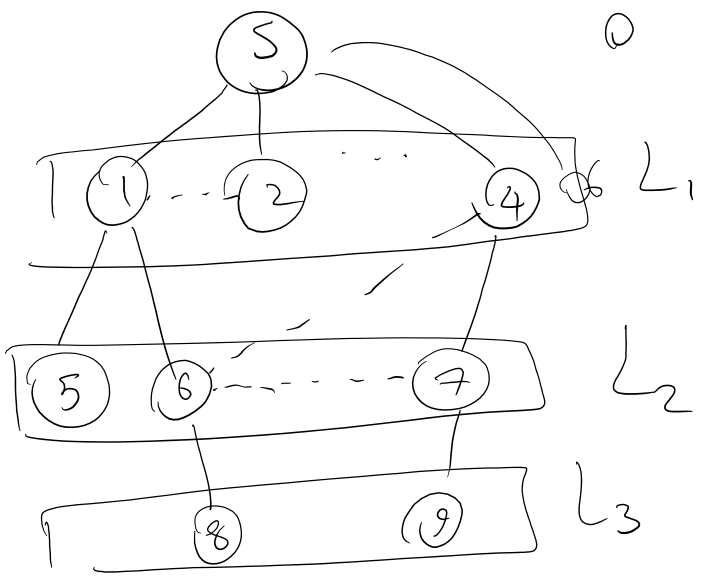
CSE 421 - Lecture 7

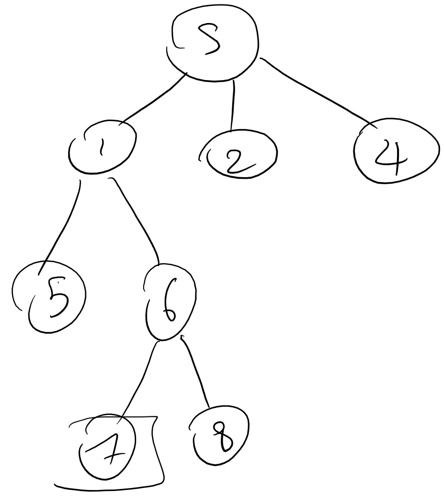
Friday, January 19, 2018 2:25 PM



queue

~~S~~, ~~1~~, ~~2~~, ~~4~~, 5, 6, 7

7 8 ~~5~~ ~~6~~ ~~1~~ 2, 4



Lemma: $L(v) = i$ iff shortes path to s is i.

pf. Cl. 1: If $L(v) = i \Rightarrow$ shor path $\leq i$.

Cl 2: If shorte path $= i \Rightarrow L(v) \leq i$

$$s = v_0, v_1, \dots, v_i = v$$

$$L(v_0) = 0$$

$$\forall j \quad L(v_j) - L(v_{j-1}) \leq 1$$

$$+\left\{\begin{array}{l} L(v_1) - L(v_0) \leq 1 \\ \vdots \\ L(v_i) - L(v_{i-1}) \leq 1 \end{array}\right.$$

$$L(v_i) - \underbrace{L(v_0)}_{0} \leq i$$

$L(v_1) - \underbrace{L(v_0)}_{0} = \cdot$

Claim: If G has at least n edges then it has a cycle

Pf: If G connected and it has no cycle $\Rightarrow$ it is a tree

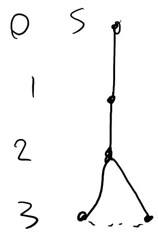
But a tree has $n-1$ edges contradiction

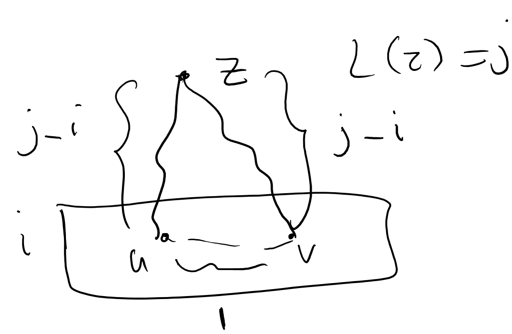
If not, $\exists$ conn comp s.t. # edges $\geq$ # vertices (because # edges in G $\geq$ # vertices in G)

so by above it has a cycle

Claim. If $L(v) = L(u) = i$ and $\{u, v\}$ an edge

then there an odd cycle.





$j - i + j - i + 1 = 2(j-i)+1$