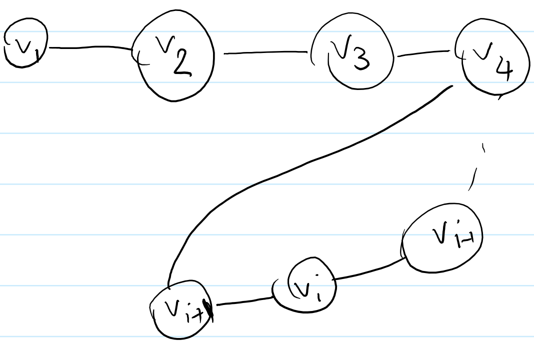
CSE 421 - Lecture 6

Wednesday, January 17, 2018 2:13 PM

Claim If G has no cycle, $\exists v$ s.t. $deg(v) \leq 1$.

pf (by contrd.)

Supp $deg(v) \geq 2$ for all v.



So continue a path: start from v_1. from v_i go to $v_{i+1} \neq v_{i-1}$

(we can do that because $deg(v_i) \geq 2$

The first time that we see repeated vertex, we get a cycle.

contradiction.

Claim: Every Tree with n vertices has n-1 edges.

Base Case n=1. Single node. n-1 edges

IH: Every tree with n-1 nodes have n-2 edges.

IS: Given a tree T with n vertices

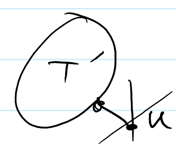
We want to show it has n-1 edges.

We proved T has a leaf u.

Remove u and its edge and call the new T'.

I claim T' is a tree.

- T' has no cycle because T has no cycle and we removed a leaf



- T' is connected because T connected and we removed a leaf.

So T' is a tree with n-1 vertices

by IH it has n-2 edges

So T has n-2+1=n-1 edges.

Claim: All non-tree edges connect vertices at the same or adjacent levels of the tree.

Pf:

Fix edge $\{x,y\}$

Say x is discovered first.

And x is at level i. $L(x)=i \Rightarrow L(y) \geq i$

Case 1: y is discovered before exploring x.

In this case $L(y) \leq i+1$ because we are still processing vertices at level i.

Case 2: y is not discov before expl x.

In this y will be discovered and $L(y)=i+1$.

Claim: All vertices at level i have shortest path i to $\underline{\underline{s}}$. (BFS(S)).

pf

If $L(v)=i$ then shortest path to $s \leq i$.

Level $i \rightarrow \exists$ path with i edges from s in BFS tree

short path $\leq i$

If shortest path $= i \Rightarrow L(v) \leq i$.