CSE 421 - Lecture 4

Wednesday, January 10, 2018 2:26 PM

Claim: f is a bijection on S.

Pf. f is one-to-one on S

So enough to show it is onto.

$\forall j \in S$, there is $i \in S$

$f(i) = j$

Because f is one-to-one it maps S to $|S|$ many values

But f maps S to S.

So f is onto

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f is onto if $\forall j$

there exist i s.t. $f(i) = j$

f is one-to-one if $\forall i, j$

$f(i) \neq f(j)$.

Pf of Correctness

By induction

Base Case: $|A| = 1 \Rightarrow S = A$

- f: maps $A \to A$ so $S \to S$
- f is one-to-one because $|S| = 1$

Ind Hyp: If f maps B to B for $|B| < n$ then we can find OPT S for B.

Ind Step: Solve for $|A| = n$.

If $\exists j$ s.t. $f(i) \neq j$ for all i.

then by claim $j \notin S$.

So OPT S is the same as OPT of $A - \{j\}$.

f maps $A - \{j\}$ to $A - \{j\}$

Otherwise S is onto.

Because f maps A to A

and it is onto it must be one-to-one