

Lecture 3 Asymptotics

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1 Asymptotics

Some properties of asymptotics:

- If $f \leq O(g)$ and $g \leq O(h)$ then $f \leq O(h)$.
- If $f \geq \Omega(g)$ and $g \geq \Omega(h)$ then $f \geq \Omega(h)$.
- If $f = \Theta(g)$ and $g = \Theta(h)$ then $f = \Theta(h)$.
- If $f = O(h)$, $g = O(h)$ then $f + g = O(h)$.

Some common running times:

- Polynomial: $O(n^d)$. Exponential $2^{O(n)}$, Logarithmic $O(\log n)$.
- For every positive ϵ (no matter how small), $\log n \leq O(n^\epsilon)$. For every positive d (no matter how large), $n^d \leq O(2^n)$.

2 In class exercise

Arrange in increasing order of asymptotic growth:

1. $f_3(n) = n^{5/3}$
2. $f_2(n) = n^{\log^2 n} \cdot 2^n$
3. $f_4(n) = n \log^3 n$
4. $f_5(n) = n^{\log n}$
5. $f_6(n) = 2^{n \log n}$.

Hint:

- Always keep in mind $n = 2^{\log_2 n}$. For example, $n^{1.5} = 2^{1.5 \log_2 n}$.
- Also recall $2^a \cdot 2^b = 2^{a+b}$.