1 Asymptotics

Some properties of asymptotics:

- If \( f \leq O(g) \) and \( g \leq O(h) \) then \( f \leq O(h) \).
- If \( f \geq \Omega(g) \) and \( g \geq \Omega(h) \) then \( f \geq \Omega(h) \).
- If \( f = \Theta(g) \) and \( g = \Theta(h) \) then \( f = \Theta(h) \).
- If \( f = O(h) \), \( g = O(h) \) then \( f + g = O(h) \).

Some common running times:

- Polynomial: \( O(n^d) \). Exponential \( 2^{O(n)} \), Logarithmic \( O(\log n) \).
- For every positive \( \epsilon \) (no matter how small), \( \log n \leq O(n^\epsilon) \). For every positive \( d \) (no matter how large), \( n^d \leq O(2^n) \).

2 In class exercise

Arrange in increasing order of asymptotic growth:

1. \( f_3(n) = n^{5/3} \)
2. \( f_2(n) = n^{\log^2 n} \cdot 2^n \)
3. \( f_4(n) = n^{\log^3 n} \)
4. \( f_5(n) = n^{\log n} \)
5. \( f_6(n) = 2^{n\log n} \).

Hint:

- Always keep in mind \( n = 2^{\log_2 n} \). For example, \( n^{1.5} = 2^{1.5\log_2 n} \).
- Also recall \( 2^a \cdot 2^b = 2^{a+b} \).