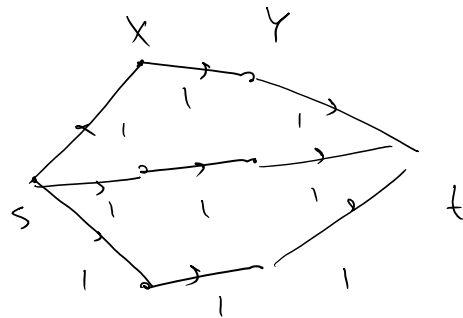
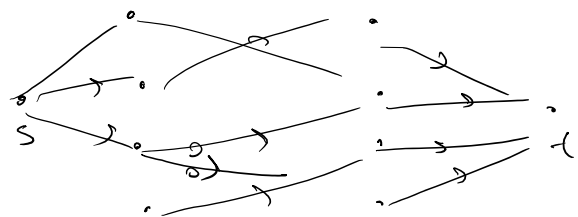


$\max \text{ mat} \leq \max \text{ flow}$



$\max \text{ mat} \geq \max \text{ flow}$

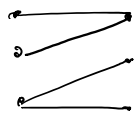


G is bipartite

$|N(S)| \geq |S| \forall S \Rightarrow G$ has a perfect matching.

Contra positive:

G has no perfect matching $\Rightarrow \exists S: |N(S)| < |S|$.



Suppose G has no perfect matching.

$\max \text{ flow} < |X|$

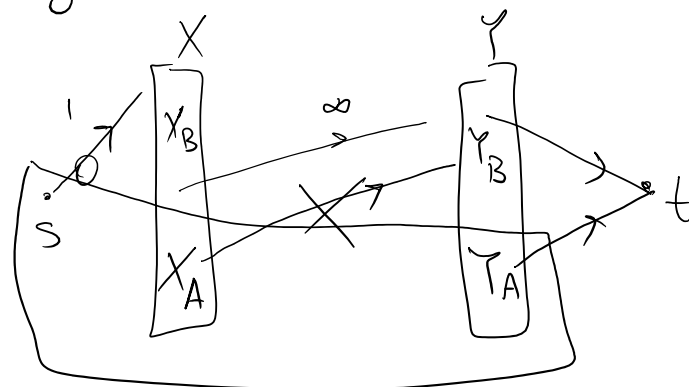
$\min \text{ cut} < |X|$

Let (A, B) be min s-t cut

$\text{cap}(A, B) < |X|$

$\text{cap}(A, B) = |X_B| + |Y_A|$

$N(X_A) \subseteq Y_A$



empty edge $X_A \rightarrow Y_B$ has ∞ capacity \Rightarrow no such edge in (A, B)

$|N(X_A)| \leq |Y_A| = \text{cap}(A, B) - |X_B| = \text{cap}(A, B) - (|X| - |X_A|) = \text{cap}(A, B) - |X| + |X_A| < |X_A|$